## LETTER

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Smoke, fog, jelly, paints, milk and shaving cream are common everyday examples of colloids  $\mathcal{L}_{\mathcal{A}}$ , and so type of soft matter consisting of  $\mathbf{r}$ tiny particles dispersed in chemically distinct host media. Being  $\hat{I}$  in a nature, colloids also find increasingly important applies also find increasingly important applications in science and technology, ranging from direct probing of kinetics in crystals and glasses $2$  to  $\tilde{R}_\text{c}$  to fabrication of third-generation of third-generation of third-generation of third-generation of the  $\tilde{R}_\text{c}$  $q=\frac{1}{2}$ . Because naturally occurring colloids have naturally occurring colloids have  $\mathbf{r}_i$ a shape that is typically determined by  $\mathcal{H}_\text{max}$  and interfacially determined by  $\mathcal{H}_\text{max}$ tension (for example, during phase separation) or faceted crystal growth1 , the interaction of the surfaces tend to have minimum-area spherical s or topologically equivalent shapes such as primarily equivalent shapes such as  $\mathbf{y}$ grains (all continuously deformable  $L$  continuously deformable  $\mathcal{S}_{\mathbf{z}}$ Although toroidal DNA condensates and vesicles with different numbers of handles can exist matter defects can be  $\mathcal{A}$ shaped as rings and knots9 , the role of particle topology in colloidal  $\mathcal{A}$ systems remains unexplored. Here we fabricate and study colloidal  $L$  and study colloidal and study colloidal and particles with different numbers of handles and genus <sup>g</sup> ranging from 1 to 5. When introduced into a nematic liquid crystal—a fluid made of rod-like molecules that spontaneously align along the socalled 'director'10—these particles induce three-dimensional director fields and topological defects dictated by colloidal topology. Whereas electric fields, photothermal melting and laser tweezing cause transformations between  $\mathcal{F}_{\mathcal{A}}$  and  $\mathcal{F}_{\mathcal{A}}$  are configurations of particle-induced induced induced induced induced induced induced in structures, three-dimensional nonlinear optical imaging reveals that topological charge is conserved and that the total charge of particle-induced defects always obeys predictions of the Gauss– Bonnet and Poincare–Hopf index theorems11–13. This allows us to establish and experimentally test the procedure for assignmental  $\ell$ and summation of topological charges in the three-dimensional direction  $\mathcal{A}$ tor fields. Our findings lay the groundwork for new applications of colloids and liquid crystals that range from topological memory devices14, through new types of self-assembly15–23, to the experimental study of low-dimensional topology6,7,11–13.

Although a coffee mug and a doughnut look different to most of us, they are topologically equivalent solid tori or handlebodies of genus  $g = 1$ , both being different from, say, balls and solid cylinders of genus  $\tilde{g} = 0$ , to which they cannot be smoothly morphed without cutting<sup>11,12</sup>. In a similar way, molecules can form topologically distinct structures including rings, knots and other molecular configurations satisfying the constraints imposed by chemical bonds<sup>24</sup>. Although the topology of shapes, fields and defects is important in many phenomena and in theories ranging from the nature of elementary particles to early-Universe cosmology<sup>25,26</sup>, topological aspects of colloidal systems (composed of particles larger than molecules and atoms but much smaller than the objects that we encounter in our everyday life) are rarely explored. Typically dealing with particles with surfaces homeomorphic to spheres, recent studies<sup>8,9,15,18,20-23</sup> demonstrate that the topology of curved surfaces dictates the formation of defects during two-dimensional colloidal crystallization at fluid interfaces as well as inside liquid crystal droplets and around spherical inclusions in liquid crystals. However, despite the fact that several techniques for scalable fabrication of particles with complex geometric shapes and g . 0 have

recently been introduced<sup>18,19,27-29</sup>, the potential impact of particle topology on colloidal alignment, self-assembly and response to fields remains unexplored.

To study the interplay of particle topology and defects in liquid crystals, we fabricated topologically distinct silica particles with planar symmetry and handlebody topology of genus g varying from 1 to 5; their surfaces had an Euler characteristic  $\chi = 2 - 2g$  ranging from 0 to  $-8$  (Fig. 1 and Supplementary Fig. 1). These particles had 1 µm  $\times$  1 µm rounded square cross-sections and ring diameters ranging from 5 to  $10 \mu m$ . Handlebody particles were introduced into a nematic liquid

into point defects and vice versa by melting the liquid crystal into an isotropic state with tweezers of laser power more than 100 mW and subsequently quenching into a nematic phase, indicating that free energies due to director configurations with these defects are comparable. These hedgehog charges of the point defects and disclination loops have been determined by assuming that the vector field lines point perpendicularly outwards from the particle surfaces (Fig. 3) and by mapping the vector fields around particles, point defects and disclination loops onto the order-parameter space<sup>31,32</sup>. Because has nonpolar symmetry (that is,  $\;$  is equivalent to  $- \;$  ), one could have chosen the vector field pointing inwards to the surface of colloids, which would consequently reverse the signs of all hedgehog charges induced by particles in a uniformly aligned liquid crystal. The relative charges of all the defects would remain the same, as would the net charge of 0,

previously observed for the 'bubble gum' configurations formed  $rac{1}{2}$  around colloidal dimers<sup>22</sup>.

We have characterized the Brownian motion of colloidal handlebodies (Fig. 4a–c). Their diffusion in a planar cell with thickness much larger than the diameter of the handlebody is highly anisotropic (Fig. 4a, d) and easier along  $_0$  than perpendicular to it. The slopes of mean square particle displacements (MSDs), shown in Fig. 4a for a solid torus (g = 1), yield diffusion coefficients  $D_x = 0.0023 \mu m^2 s^{-1}$ and  $D_y = 0.0034 \,\mathrm{\upmu m^2 s^{-1}}$  measured normal and parallel to  $_{0}$ , respectively. Being oriented with respect to  $_0$  (Fig. 4d), particles also experience angular thermal fluctuations (Fig. 4b) with  $\langle \Delta \theta^2 \rangle$  of angular displacements ( $MSD<sub>θ</sub>$ ) initially increasing linearly with the lag time  $\tau$  and then saturating as a result of the elasticity-mediated alignment. The width of the histogram distribution of the angle  $\theta$  between the axis of revolution of the solid torus and  $_0$  (Fig. 4b, inset) is  $9.6 \times 10^{-3}$ 

substrates (Fig. 4e). However, because of slow rotation of the ring compared with the roughly 10-ms response time of  $($   $)$ , an abrupt application of simply alters ( ) around the particle while preserving the initial particle alignment in the cell. For a solid torus ( $g = 1$ ), this causes the original () to transform into a topologically equivalent configuration with two disclination loops (Fig. 4f, g). Using different voltage-driving schemes, colloidal handlebodies and structures around them can be switched between the two bistable orientations and ( ) configurations shown in Fig 4d, f that are stable at no applied field. All observed transformations of (