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Labor Hoarding, Superior Information, and Business Cycle Dynamics

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1. Introduction

The objective of real business cycle (RBC) studies is to evaluate the ability of dynamic, stochastic, general equilibrium artificial economies to account for observed business cycle facts. This evaluation proceeds as follows. First, the RBC researcher specifies a set of market features (e.g. preferences, technology, and market clearing conditions) that describes the environment in which agents form their decisions. These decisions rely on forward-looking rules. The researcher also specifies a law of motion for the forcing variables that depict the stochastic nature of the artificial economy. This law of motion is required to forecast the future forcing variables involved in the decision rules. Second, the researcher characterizes the equilibrium allocation of some key variables. As such equilibrium usually does not possess an analytical solution, the allocation is approximated using numerical methods. This approximation requires that values be assigned to all parameters. Generally, the researcher calibrates the parameters underlying the market features using long-run averages and previously published estimates. The researcher also calibrates the parameters of the law of motion for forcing variables from estimates on historical data. Finally, the researcher assesses the ability of its artificial economy to account for business cycle facts by confronting certain statistics computed from the artificial economy to those found in historical data.

This evaluation is thus a joint test of the calibrated market features and of the estimated law of motion. In this context, the business cycle statistics computed from the artificial economy may not match the data, not because market features are inadequately described, but simply because the law of motion is misspecified. In RBC studies, the standard law of motion involves only forcing variables. This presumes that the relevant information set used by economic agents to forecast future forcing variables includes exclusively the history of forcing variables. It seems most likely, however, that the law of motion

business cycle fluctuations for output, consumption, and investment. Also, we confront the business cycle properties predicted by the labor-hoarding environment with a basic law of motion to those predicted with augmented laws of motion. This evaluation is important macroeconomic aggregates. Hence, the labor-hoarding environment with a basic law of motion fails to reproduce observed business cycle facts.

In contrast, our results show that the labor-hoarding environment with augmented laws of motion tracks remarkably well observed volatility, cross-correlations, and dynamic responses. These findings hold for all the key macroeconomic aggregates and both measures of the cycle. Hence, the labor-hoarding environment with augmented laws of motion successfully matches observed business cycle facts.

The paper is organized as follows. Section 2 presents the labor-hoarding environment, its calibration, and its solution with basic and augmented laws of motion for forcing variables. Section 3 documents empirical results for the growth rates of output, consumption, and investment. Section 4 discusses results for the cyclical components of our macroeconomic aggregates. Section 5 concludes.

2. The Economic Environment

Our analysis is based on both unrestricted and restricted vector autoregressions (VARs) for selected variables. As in most of the RBC literature, these VARs are obtained using calibrated decision rules and estimated laws of motion for forcing variables. In what follows,

nonpredetermined and predetermined (state) variables:

$$\widehat{\mathbf{m}}_t \equiv \mathbf{m}_t - \theta_{11}\mathbf{p}_t = \theta_{12}\mathbf{s}_t + \theta_{13}E_t\left[\sum_{j=1}^{\infty}\lambda^{-j}\mathbf{s}_{t+j}\right], \tag{2.1}$$

$$\widehat{\mathbf{p}}_{t+1} \equiv \mathbf{p}_{t+1} - \theta_{21}\mathbf{p}_t = \theta_{22}\mathbf{s}_t + \theta_{23}E_t\left[\mathbf{s}_{t+1}\right] + \theta_{24}E_t\left[\sum_{j=1}^{\infty}\lambda^{-j}\mathbf{s}_{t+j}\right], \tag{2.2}$$

where $\mathbf{m}_t = (\, y_t \quad c_t \quad i_t\,)'$ is the vector of nonpredetermined variables, $\mathbf{p}_t = (\, k_t \quad n_t\,)'$ is the vector of predetermined variables, and $\mathbf{s}_t = (\, z_t \quad g_t\,)'$ is the vector of forcing variables. The variables are $y_t = \ln(\widetilde{Y}_t/\widetilde{Y})$ for $\widetilde{Y}_t = Y_t/Z_t$, $c_t = \ln(\widetilde{C}_t/\widetilde{C})$ for $\widetilde{C}_t = C_t/Z_t$, $i_t = \ln(\widetilde{I}_t/\widetilde{I})$ for $\widetilde{I}_t = I_t/Z_t$, $k_{t+1} = \ln(\widetilde{\kappa}_{t+1}/\widetilde{\kappa})$ for $\widetilde{\kappa}_{t+1} = \kappa_{t+1}/Z_t$, $n_{t+1} = \ln(N_{t+1}/N)$, $z_t = \ln(\widetilde{Z}_t/\widetilde{Z})$ for $\widetilde{Z}_t = Z_t/Z_{t-1}$, and $g_t = \ln(\widetilde{G}_t/\widetilde{G})$ for $\widetilde{G}_t = G_t/Z_t$—where $\widetilde{Y}$, $\widetilde{C}$, $\widetilde{I}$, $\widetilde{\kappa}$, N, $\widetilde{Z}$, and $\widetilde{G}$ are the steady state values. The parameters λ and θs are complex functions of the underlying parameters as well as $\widetilde{Z}$ and $\widetilde{G}/\widetilde{Y}$. We obtain values for them using the calibration of Boileau and Normandin (2001) and Burnside and Eichenbaum (1996): $\beta = 1.03^{-0.25}$, $\eta = 3.89$, $H = 1369$, $\zeta = 60$, $\quad = 324.8$, $\delta = 0.021$, and $\alpha = 0.344$. We also use a sample of U.S. seasonally adjusted quarterly data over the 1960:II to 1993:IV period to set $\widetilde{Z} = 1.0031$ and $\widetilde{G}/\widetilde{Y} = 0.125$ (see Data Appendix).

2.2 The Basic Law of Motion

To obtain the restricted VARs, decision rules (2) must be expressed exclusively in terms of observed variables. This requires a law of motion to construct the expectations of future forcing variables that appear in decision rules (2). As a benchmark, we posit the law of motion:[1]

$$\begin{pmatrix} z_t \\ g_t \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \rho \end{pmatrix}\begin{pmatrix} z_{t-1} \\ g_{t-1} \end{pmatrix} + \begin{pmatrix} u_{zt} \\ u_{gt} \end{pmatrix}$$

where $\Omega_s = E[\mathbf{u}_t \mathbf{u}_t']$. This basic law of motion implies that the relevant information set used to forecast future forcing variables contains exclusively the history of forcing variables.

An estimated version of this law of motion is used to construct the expectations of future forcing variables. OLS estimation yields $\rho = 0.969$, $E[u_{zt}u_{zt}] = 0.000084$, $E[u_{gt}u_{gt}] = 0.000689$, and $E[u_{zt}u_{gt}] = -0.000048$. The constructed expectations are then substituted in decision rules (2) to yield:

$$\widehat{\mathbf{m}}_t^b = \varphi_{ms}\mathbf{s}_t, \tag{4.1}$$

$$\widehat{\mathbf{p}}_{t+1}^b = \varphi_{ps}\mathbf{s}_t. \tag{4.2}$$

The parameters φs are functions of the calibrated parameters in (2) and the estimated parameters in (3): $\varphi_{ms} = \theta_{12} + \theta_{13}\left[I_s - \lambda^{-1}\Pi_s\right]^{-1}\lambda^{-1}\Pi_s$ and $\varphi_{ps} = \theta_{22} + \theta_{23}\Pi_s + \theta_{24}\left[I_s - \lambda^{-1}\Pi_s\right]^{-1}\lambda^{-1}\Pi_s$, where I_s is an identity matrix. The superscript b indicates that these are predicted variables using the basic law of motion.

The restricted VARs are built from the basic law of motion (3) and reduced forms (4). For example, we obtain a restricted VAR for output as follows. First, we write the reduced form for output contained in (4.1) as:

$$\widehat{y}_t^b = \varphi_{yz}z_t + \varphi_{yg}g_t, \tag{5}$$

where φ_{yz} and φ_{yg} are the elements on the first line of φ_{ms}. Then, we use the law of motion (3) to produce:

$$\begin{pmatrix} z_t \\ g_t \\ \widehat{y}_t^b \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \rho & 0 \\ 0 & \phi & 0 \end{pmatrix} \begin{pmatrix} z_{t-1} \\ g_{t-1} \\ \widehat{y}_{t-1}^b \end{pmatrix} + \begin{pmatrix} u_{zt} \\ u_{gt} \\ u_{yt}^b \end{pmatrix}$$

or

$$\mathbf{x}_{yt}^b = \Phi_y^b \mathbf{x}_{yt-1}^b + \mathbf{u}_{yt}^b, \tag{6}$$

where $\phi = \varphi_{yg}\rho$ and $u_{yt}^b = \varphi_{yz}u_{zt} + \varphi_{yg}u_{gt}$. Using this method, we obtain restricted VARs for all nonpredetermined and predetermined variables.

2.3 Augmented Laws of Motion

The basic law of motion (3) imposes the restrictive assumption that the relevant information set that agents use to forecast future forcing variables includes only the history of those forcing variables. It seems plausible, however, that agents possess extra relevant information to construct these forecasts. In what follows, we assume that this extra information is embodied in a single hidden variable h

and Normandin (2001). This method allows us to obtain laws of motion and associated reduced forms that contain only observables. Under the null hypothesis that decision rules (2) are valid, reduced forms (8) indicate that agents reveal their expectations of future forcing variables through their decisions about $\widehat{\mathbf{m}}_t$ and $\widehat{\mathbf{p}}_{t+1}$. Then, an adequate law of motion for forcing variables is obtained by replacing the hidden variable h_t by any variable included in either $\widehat{\mathbf{m}}_t$ or $\widehat{\mathbf{p}}_{t+1}$. This result in a law of motion and reduced forms that are augmented by agents' superior information.

We illustrate this procedure using output. First, we write the reduced form for output contained in (8.1) as:

$$\widehat{y}_t = \vartheta_{yz} z_t + \vartheta_{yg} g_t + \vartheta_{yh} h_t, \tag{9}$$

where ϑ_{yz}, ϑ_{yg}, and ϑ_{yh} are the elements on the first line of ϑ_{mw}. Second, we rewrite this reduced form as:

$$\begin{pmatrix} z_t \\ g_t \\ \widehat{y}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vartheta_{yz} & \vartheta_{yg} & \vartheta_{yh} \end{pmatrix} \begin{pmatrix} z_t \\ g_t \\ h_t \end{pmatrix}$$

or

$$\mathbf{x}_{yt} = \Upsilon_y \mathbf{w}_t. \tag{10}$$

Third, we substitute (7) in (10) to obtain a VAR for output:

$$\begin{pmatrix} z_t \\ g_t \\ \widehat{y}_t \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix} \begin{pmatrix} z_{t-1} \\ g_{t-1} \\ \widehat{y}_{t-1} \end{pmatrix} + \begin{pmatrix} u_{zt} \\ u_{gt} \\ u_{yt} \end{pmatrix}$$

or

$$\mathbf{x}_{yt} = \Gamma_y \mathbf{x}_{yt-1} + \mathbf{u}_{yt}, \tag{11}$$

where $\Gamma_y = \Upsilon_y \Pi_w \Upsilon_y^{-1}$ and $\mathbf{u}_{yt} = \Upsilon_y \mathbf{v}_t$. The first two equations of (11) form the law of motion for forcing variables augmented by output. In this augmented law of motion, the feedbacks from lagged $\widehat{y}_t$ to current forcing variables reflect the effects of the lagged h_t on contemporaneous forcing variables highlighted in the true law of motion (7). The last equation of (11) states that the innovation of output is a function of the innovations of forcing and hidden variables: $u_{yt} = \vartheta_{yz} u_{zt} + \vartheta_{yg} u_{gt} + \vartheta_{yh} u_{ht}$. This formulation is in accord with the notion that $\widehat{y}_t$ fully summarizes the relevant information.

Fourth, given that the augmented law of motion contains all the relevant information, we estimate an unrestricted VAR for output. OLS estimation yields $\gamma_{11} = 0.333$, $\gamma_{12} = 0.015$, $\gamma_{13} = 0.012$, $\gamma_{21} = -0.226$, $\gamma_{22} = 0.971$, $\gamma_{23} = -0.064$, $\gamma_{31} = 0.078$, $\gamma_{32} = 0.001$, and $\gamma_{33} = 1.013$. It also yields $E\left[u_{zt} u_{zt}\right] = 0.000070$, $E\left[u_{gt} u_{gt}\right] = 0.000686$, $E\left[u_{yt} u_{yt}\right] = 0.000023$, $E\left[u_{zt} u_{gt}\right] = -0.000042$, $E\left[u_{zt} u_{yt}\right] = -0.000035$, and $E\left[u_{gt} u_{yt}\right] = 0.000030$.[3] The constructed expectations are then substituted in the decision rule for output contained in (2.1) to yield:

$$\widehat{y}^a_t = \varphi_y \; E[\quad u_{gt} u_{yt} \hat{z}_t + \sum \; [\; a \; \sum \; t \; [\; t \sum t \; g \; t + \; \sum \; \beta \; \beta$$

or

$$\mathbf{x}^a_{j,t} = \Phi^a$$

$$a$$

predicted to observed measures is unity. For this test, we treat the observed measure as a constant and the predicted one as a random variable, where the variance of the predicted measure accounts for the uncertainty of the estimated parameters in either basic or augmented laws of motion.

Table 1 confronts observed and predicted measures of volatility. For output, the volatility predicted with augmented laws of motion is both statistically and numerically closer to the observed volatility. The sample estimate for $\sigma_{\Delta \ln(Y)}$ is 0.856 percent. The volatility (p-value) predicted with the basic law of motion is 0.787 percent (0.000). Accordingly, the ratio of predicted to observed volatility is 91.9 percent and statistically different from unity. The volatility (p-value) predicted with augmented laws of motion, however, is 0.825 percent (0.813). The ratio of predicted to observed volatility is 96.4 percent and insignificantly different from unity.

For consumption, the volatility predicted with either the basic or augmented laws of motion is numerically and statistically close to the observed volatility. The sample estimate for $\sigma_{\Delta \ln(C)}$ is 0.512 percent. The volatility (p-value) predicted by the environment with the basic law of motion is 0.488 percent (0.229) and the volatility (p-value) predicted with augmented laws of motion is 0.538 percent (0.879). This pattern extends to the measure

augmented laws of motion, however, generally replicates them.

3.2. Dynamic Responses

We now study the dynamic responses of the growth rates of output, consumption, and investment to both positive technology and government expenditure growth shocks. The observed responses are computed using the identities $\Delta \ln(Y_t) = \Delta y_t + z_t + \ln(\widetilde{Z})$, $\Delta \ln(C_t) = \Delta c_t + z_t + \ln(\widetilde{Z})$, $\Delta \ln(I_t) = \Delta i_t + z_t + \ln(\widetilde{Z})$, and $\Delta \ln(G_t) = \Delta g_t + z_t + \ln(\widetilde{Z})$, as well as the appropriate unrestricted VARs for nonpredetermined and predetermined variables. For example, the unrestricted VAR for output is similar to (11):

$$\mathbf{x}_{\boldsymbol{y}t} = \Gamma_{\boldsymbol{y}} \mathbf{x}_{\boldsymbol{y}t-1} + \mathbf{u}_{\boldsymbol{y}t}. \tag{15}$$

The predicted responses are computed from the relevant identities and restricted VARs (see Technical Appendix). We also calculate the p-value from a $\chi^2(1)$ distributed statistic of the test that the difference between predicted and observed responses is null. For this test, we treat the observed response as a constant and the predicted one as a random variable, where the variance of the predicted response accounts for the uncertainty of the estimated parameters in either the basic or augmented laws of motion.

Figures 1 and 2 depict observed and predicted dynamic responses. For output, the observed responses to both technology and government expenditure growth shocks exhibit an increase at impact followed by a decay. The dynamic responses predicted by the environment with either basic or augmented laws of motion numerically replicate this pattern well. The responses generated with the basic law of motion, however, are almost always significantly different from observed ones. With the exception of the first quarter, the responses generated with augmented laws of motion are insignificantly different from observed responses.

For consumption, the observed and predicted responses to both technology and government expenditure growth shocks display an increase at impact, followed by a second peak, and then a decay. As for output, the responses generated with the basic law of motion are most of the time significantly different from observed responses, while responses generated with augmented laws of motion are almost always insignificantly different from observed ones.

Finally, for investment, the observed responses to a technology growth shock show a large increase at impact, followed by a trough and a return to the steady state. The observed responses to a government expenditure growth shock show a small increase at impact, followed by a peak, a trough, and a gradual return to the steady state. The responses to both shocks predicted with the basic law of motion display a large increase at impact, followed by a trough and a return to the steady state. With one exception, these responses are significantly different from observed ones. The responses to a technology growth shock predicted with augmented laws of motion exhibit a large increase at impact followed by a peak, a trough, and a return to the steady state. Except for the first quarter, these responses are insignificantly different from observed responses. The responses to a government expenditure growth shock display a reduction at impact, followed by a peak, a trough, and a return. These responses are often insignificantly different from observed responses.

Thus, the predicted responses computed with both the basic and augmented laws of motion predict observed dynamics fairly well. However, responses predicted with the basic law of motion are significantly different from observed responses, while those generated with augmented laws of motion are insignificantly different from observed responses.

4. Results: Cyclical Components

In this section, we evaluate whether the labor-hoarding environment with basic and augmented laws of motion account for business cycle facts, when the cycle component of a series corresponds to the cycle definition of Beveridge and Nelson (1981). We show that the labor-hoarding environment with the basic law of motion mostly fails to reproduce observed business cycle facts, while the environment with augmented laws of motion almost always replicates them.

4.1 Volatility and Cross-Correlation

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and numerically close to the observed volatility. This observed volatility $\sigma_{\ln(Y)^c}$ is 0.133. The volatility (p-value) predicted with the basic law of motion is 0.013 (0.000). Accordingly, the ratio of predicted to observed volatility is only 9.8 percent and significantly different from unity. The volatility (p-value) predicted with augmented laws of motion, however, is 0.124 (0.973). The ratio of predicted to observed volatility is 93.2 percent and insignificantly different from unity.

For consumption, the volatility predicted with augmented laws of motion is both statistically and numerically closer to the observed volatility. The observed volatility $\sigma_{\ln(C)^c}$ is 0.290. The volatility (p-value) predicted by the environment with the basic law of motion is 0.016 (0.000) and that predicted with augmented laws of motion is 0.175 (0.808). However, for $\sigma_{\ln(C)^c}/\sigma_{\ln(Y)^c}$, the environments with both basic and augmented laws of motion predict a relative volatility close to the observed one. This occurs because the environment with the basic law of motion undervalues the volatility for both output and consumption.

Finally, for investment, augmented laws of motion also outperform the basic law of motion. The observed volatility $\sigma_{\ln(I)^c}$

of output, consumption, and investment to both positive technology and government expenditure growth shocks. The observed responses are computed using the cycle definition (16), as well as the appropriate unrestricted VARs for nonpredetermined and predetermined variables. The predicted responses are computed from the relevant definition and restricted VARs (see Technical Appendix). As before, we also calculate the p-value from a $\chi^2(1)$ distributed statistic of the test that the difference between predicted and observed responses is null.

Figures 3 and 4 display observed and predicted dynamic responses. For output, the observed responses to a technology growth shock show an increase at impact and an hump-shaped return. The observed responses to a government expenditure growth shock exhibit a reduction at impact and a gradual return to the steady state. The responses predicted with the basic law of motion display the wrong sign at impact and substantially understate observed responses. The predicted responses to a technology growth shock show a small decrease at impact and a rapid return to the steady state, while the predicted responses to a government expenditure growth shock show a slight increase at impact followed by a hump-shaped return. Clearly, for both shocks, the differences between predicted and observed responses are always significantly different from zero. The dynamic responses predicted with augmented laws of motion, however, track observed responses extremely well, such that the differences between predicted and observed responses are always insignificantly different from zero.

For consumption, the observed responses to a technology growth shock display an increase at impact followed by a further increase after three quarters. The observed responses to a government expenditure growth shock exhibit a reduction at impact followed by a return to the steady state. The responses predicted with the basic law of motion significantly undervalue these responses. As for output, the responses predicted with augmented laws of motion are always insignificantly different from observed responses.

Finally, for investment, the observed responses to both technology and government expenditure growth shocks show a large increase at impact, followed by a reduction and a return to the steady state. The responses to both shocks predicted with the basic law of motion display a small increase at impact, followed by a hump-shaped return to the steady state. Once again, these responses are significantly smaller than the observed ones. In contrast, the responses predicted with augmented laws of motion are always insignificantly different from observed responses.

Hence, the predicted responses computed with the basic law of motion greatly undervalue observed responses. This accords with our empirical results that predicted measures of volatility grossly understate observed measures. The responses predicted with augmented laws of motion, however, replicate observed dynamic responses remarkably well.

5. Conclusion

In this paper, we test whether an artificial economy with labor hoarding provides an adequate explanation for observed business cycle dynamics. Importantly, our evaluation is performed using two different descriptions for the law of motion of the economy's forcing variables. The first assumes that the information set used to forecast future forcing variables contains exclusively the history of these forcing variables. This leads to a basic law of motion that only includes forcing variables. The second assumes that the relevant information set is superior and includes not only forcing variables but also hidden exogenous variables. This leads to augmented laws of motion that include both forcing and endogenous variables, where the endogenous variables replace hidden variables.

We show that omitting the hidden variables leads to serious mismeasurements of the business cycle. More precisely, we find that the labor-hoarding environment with a basic law of motion predicts volatility, cross-correlations, and dynamic responses of key macroeconomic aggregates that substantially deviate from observed ones. This holds but it is especially severe for the latter. Hence, the labor-hoarding environment with a basic law of motion fails to reproduce observed business cycle facts. In contrast, our

Technical Appendix

The VARs

With the exception of the observed measures of volatility, all our computations are based on VARs. The unrestricted VARs for output, consumption, investment, employment, and or

$$\mathbf{X}_t^r = \mathbf{\Phi}^r \mathbf{X}_{t-1}^r + \mathbf{U}_t^r. \tag{A.4}$$

The covariance matrices are $\mathbf{\Omega}^r = E\left[\mathbf{U}_t^r \mathbf{U}_t^{r\prime}\right]$ and $\mathbf{\Sigma}^r = E\left[\mathbf{X}_t^r \mathbf{X}_t^{r\prime}\right]$, where $\mathbf{\Sigma}^r = \mathbf{\Phi}^r \mathbf{\Sigma}^r \mathbf{\Phi}^{r\prime}$ $+\mathbf{\Omega}^r$.

Volatility and Correlation of Growth Rates

The observed measures of volatility and correlation for the growth rates of output, consumption, and investment are computed from sample estimates of variances and covariances in our U.S. quarterly data. Variances are computed as:

$$\sigma_x^2 = \frac{1}{T}\sum_{t=1}^{T}\left(x_t - \overline{x}\right)^2, \tag{A.5}$$

where $\overline{x} = (1/T)\sum_{t=1}^{T} x_t$ for $x_t = \Delta\ln(Y_t)$, $\Delta\ln(C_t)$, $\Delta\ln(I_t)$. Covariances are computed as:

$$\text{cov}[x_t, \Delta\ln(Y_{t+k})] = \frac{1}{T-|k|}\sum_{t=1}^{T-|k|}\left(x_t - \overline{x}\right)\left(\Delta\ln(Y_{t+k}) - \overline{\Delta\ln(Y)}\right), \tag{A.6}$$

for $x_t = \Delta\ln(C_t)$, $\Delta\ln(I_t)$.

The predicted measures of volatility and correlation are computed from the stacked VAR ($A.4$). First, we use the relations $\Delta\ln(Y_t) = \Delta y_t + z_t + \ln(\widetilde{Z})$, $\Delta\ln(C_t) = \Delta$

where $\text{vec}[\Omega^r_{01}] = \left((\theta_{21}e_p \otimes \mathbf{\Phi}^r)' \otimes (\Psi^r_2 \otimes \Psi^r_2)\right) \left[I_{p^3q} - (\theta_{21} \otimes I_q)' \otimes (I_p \otimes \theta_{21})\right]^{-1}$ $\text{vec}\left[(I_p \otimes e_p)\left[I_{pq} - \theta_{21} \otimes \mathbf{\Phi}^r\right]^{-1}\right]$ and $\text{vec}[\Omega^r_{02}] = \left((\mathbf{\Phi}^r \otimes \theta_{21}e_p)' \otimes (\Psi^r_2 \otimes \Psi^r_2)\right)$ $\left[I_{p^3q} - (I_q \otimes \theta_{21})' \otimes (\theta_{21} \otimes I_p)\right]^{-1}$ $\text{vec}\left[(e_p \otimes I_p)\left[I_{pq} - \mathbf{\Phi}^r \otimes \theta_{21}\right]^{-1}\right]$. Finally, I_p, I_q, I_{p^2}, I_{pq}, and I_{p^3q} are identity matrices, where p and q refer to the dimensions of $\mathbf{p}_t$ and $\mathbf{X}_t$.

Finally, for $k = 1$, 2, and 4, we compute:

$$\begin{aligned}
E\left[\mathbf{W}^{rg}_t \mathbf{W}^{rg\prime}_{t-k}\right] =& (\Psi^r_0 \otimes B_k)\text{vec}[\mathbf{\Sigma}^r] + (\Psi^r_1 \otimes \Psi^r_2\theta^{k-1}_{21}e_p)\text{vec}[\mathbf{\Sigma}^r] + \\
& (\Psi^r_2 \otimes \Psi^r_2)\left[I_{p^2} - \theta_{21} \otimes \theta_{21}\right]^{-1}(e_p \otimes \theta^k_{21}e_p)\text{vec}[\mathbf{\Sigma}^r] + \\
& (\Psi^r_1 \otimes B_k\mathbf{\Phi}^r)\text{vec}[\mathbf{\Sigma}^r] + \\
& (\Psi^r_2 \otimes B_k)\left[I_{pq} - \theta_{21} \otimes \mathbf{\Phi}^r\right]^{-1}(e_p \otimes \mathbf{\Phi}^{r2})\text{vec}[\mathbf{\Sigma}^r] + \\
& \Omega^r_{k1}\text{vec}[\mathbf{\Sigma}^r] + (\Psi^r_0 \otimes \Psi^r_2)\left[I_{pq} - \mathbf{\Phi}^r \otimes \theta_{21}\right]^{-1}(\mathbf{\Phi}^r \otimes \theta^{k-1}_{21}e_p)\text{vec}[\mathbf{\Sigma}^r] + \\
& (\Psi^r_1 \otimes \Psi^r_2)\left[I_{pq} - \mathbf{\Phi}^r \otimes \theta_{21}\right]^{-1}(\mathbf{\Phi}^r \otimes \theta^k_{21}e_p)\text{vec}[\mathbf{\Sigma}^r] + \Omega^r_{k2}\text{vec}[\mathbf{\Sigma}^r] \quad (A.9)
\end{aligned}$$

where $\text{vec}[\Omega^r_{k1}] = \left((e_p \otimes \mathbf{\Phi}^r)' \otimes (\Psi^r_2 \otimes \Psi^r_2)\right) \left[I_{p^3q} - (\theta_{21} \otimes I_q)' \otimes (I_p \otimes \theta_{21})\right]^{-1}$ $\text{vec}\left[(I_p \otimes \theta^{k-1}_{21}e_p)\left[I_{pq} - \theta_{21} \otimes \mathbf{\Phi}^r\right]^{-1}\right]$ and $\text{vec}[\Omega^r_{k2}] = \left((\mathbf{\Phi}^r \otimes \theta^{k+1}_{21}e_p)' \otimes (\Psi^r_2 \otimes \Psi^r_2)\right)$ $\left[I_{p^3q} - (I_q \otimes \theta_{21})' \otimes (\theta_{21} \otimes I_p)\right]^{-1}$ $\text{vec}\left[(e_p \otimes I_p)\left[I_{pq} - \mathbf{\Phi}^r \otimes \theta_{21}\right]^{-1}\right]$. Finally, $B_1 = \Psi^r_0\mathbf{\Phi}^r + \Psi^r_1$, $B_2 = \Psi^r_0\mathbf{\Phi}^{r2} + \Psi^r_1\mathbf{\Phi}^r + \Psi^r_2e_p$, and $B_4 = \Psi^r_0\mathbf{\Phi}^{r4} + \Psi^r_1\mathbf{\Phi}^{r3} + \Psi^r_2e_p\mathbf{\Phi}^{r2} + \Psi^r_2\theta_{21}e_p\mathbf{\Phi}^r + \Psi^r_2\theta^2_{21}e_p$.

Dynamic Responses of Growth Rates

The observed dynamic responses are computed using the unrestricted VARs, as well as the relations $\Delta \ln($ f G

where $e_1 = (1 \quad 0 \quad 0)$, $e_2 = (0 \quad 1 \quad 0)$, and $e_3 = (0 \quad 0 \quad 1)$. Also, θ_{yn} and θ_{yk} are the elements on the first line of θ_{11}, θ_{nn} and θ_{nk} are the elements on the first line of θ_{21}, and θ_{kn} and θ_{kk} are the elements on the second line of θ_{21}. Note that $\Omega_{yy} = \Lambda_y \Lambda_y{}'$, $\Omega_{nn} = \Lambda_n \Lambda_n{}'$, and $\Omega_{kk} = \Lambda_k \Lambda_k{}'$, where Λ_y, Λ_n, and Λ_k are lower triangular matrices with positive elements on their diagonals. This allows us to extract orthogonal innovations $\mathbf{e}_{yt} = \Lambda_y^{-1}\mathbf{u}_{yt}$, where $\mathbf{e}_{yt} = (e_{zt} \quad e_{gt} \quad e_{yt})'$. The technology growth shock is measured by a positive one standard deviation in e_{zt}, $\bar{e} = e_1$. The government expenditure growth shock is measured as the sum of a positive one standard deviation in e_{zt} and e_{gt}, $\bar{e} = e_1 + e_2$, since $\Delta \ln(G_t) = \Delta g_t + z_t + \ln(\widetilde{Z})$.

Similarly, the predicted dynamic responses are constructed using the restricted VARs and relevant relations and identities. For output growth, we first compute the responses of y_t, z_t, n_{t+1}, and k_{t+1} using the appropriate restricted VARs:

$$R^r_{y,j} = e_3 \Phi_y^{r\,j} \Lambda_y^r \bar{e}' + \theta_{yn} R^r_{n,j-1} + \theta_{yk} R^r_{k,j-1}, \tag{A.15}$$

$$R^r_{z,j} = e_1 \Phi_y^{r\,j} \Lambda_y^r \bar{e}', \tag{A.16}$$

$$R^r_{n,j} = e_3 \Phi_n^{r\,j} \Lambda_n^r \bar{e}' + \theta_{nn} R^r_{n,j-1} + \theta_{nk} R^r_{k,j-1}, \tag{A.17}$$

$$R^r_{k,j} = e_3 \Phi_k^{r\,j} \Lambda_k^r \bar{e}' + \theta_{kn} R^r_{n,j-1} + \theta_{kk} R^r_{k,j-1}. \tag{A.18}$$

Then, we compute the predicted responses as:

$$R^r_{\Delta \ln(Y),j} = R^r_{y,j} - R^r_{y,j-1} + R^r_{z,j}. \tag{A.19}$$

Note that $\Omega^r_{yy} = \Lambda^r_y \Lambda^r_y{}'$, $\Omega^r_{nn} = \Lambda^r_n \Lambda^r_n{}'$, and $\Omega^r_{kk} = \Lambda^r_k \Lambda^r_k{}'$, where Λ^r_y, Λ^r_n, and Λ^r_k are lower triangular matrices with positive elements on their diagonals.

Volatility and Correlation of Cyclical Components

The observed measures of volatility and correlation for the cyclical components of the logarithms of output, consumption, and investment are computed from the stacked VAR (A.2) using the cycle definition (16). First, we use the relations $\ln(Y_{t+h}) - \ln(Y_t) = y_{t+h} - y_t + \sum_{j=1}^{h} z_{t+j} + h\ln(\widetilde{Z})$, $\ln(C_{t+h}) - \ln(C_t) = c_{t+h} - c_t + \sum_{j=1}^{h} z_{t+j} + h\ln(\widetilde{Z})$, and $\ln(I_{t+h}) - \ln(I_t) = i_{t+h} - i_t + \sum_{j=1}^{h} z_{t+j} + h\ln(\widetilde{Z})$ to define the vector of cyclical components $\mathbf{W}^c_t = -\lim_{h\to\infty} E_t\left[\mathbf{m}_{t+h} - \mathbf{m}_t + e'\sum_{j=1}^{h} z_{t+j}\right] = \mathbf{m}_t - e'\sum_{j=1}^{\infty} E_t\left[z_{t+j}\right]$, where $\mathbf{W}^c_t = (\ln(Y_t)^c \quad \ln(C_t)^c \quad \ln(I_t)^c)'$. Second, we employ the identities $\mathbf{m}_t = \widehat{\mathbf{m}}_t + \theta_{11}\mathbf{p}_t$ and $\mathbf{p}_{t+1} = \widehat{\mathbf{p}}_{t+1} + \theta_{21}\mathbf{p}_t$ to write $\mathbf{W}^c_t$ in terms of $\mathbf{X}_t$:

$$\mathbf{W}^c_t = \Psi_0 \mathbf{X}_t + \Psi_1 \mathbf{X}_{t-1} + \Psi_2 \sum_{j=0}^{\infty} \theta_{21}^j e_p \mathbf{X}_{t-2-j}, \tag{A.20}$$

where $\Psi_0 = e_m + e' e_z \left[I_q - \mathbf{\Gamma}\right]^{-1} \mathbf{\Gamma} - \theta_{11} e_p$, $\Psi_1 = -\theta_{11}\theta_{21} e_p$, and $\Psi_2 = -\theta_{11}\theta_{21}^2$. Finally, we use $(A.20)$ to compute the necessary moments of $\mathbf{W}_t^c$. These computations use equations similar to $(A.8)$ and $(A.9)$.

Similarly, the predicted moments are computed from the stacked VAR $(A.4)$. We use the required relations and identities to write the vector of predicted cyclical components in terms of $\mathbf{X}_t^r$:

$$\mathbf{W}_t^{rc} = \Psi_0^r \mathbf{X}_t^r + \Psi_1^r \mathbf{X}_{t-1}^r + \Psi_2^r \sum_{j=0}^{\infty} \theta_{21}^j e_p' \mathbf{X}_{t-2-j}^r, \tag{A.21}$$

where $\Psi_0^r = e_m + e' e_z \left[I_q - \mathbf{\Phi}^r\right]^{-1} \mathbf{\Phi}^r - \theta_{11} e_p$, $\Psi_1^r = -\theta_{11}\theta_{21} e_p$, and $\Psi_2^r = -\theta_{11}\theta_{21}^2$. Then, we use $(A.21)$ to compute the necessary moments of $\mathbf{W}_t^{rc}$, where the computations use equations similar to $(A.8)$ and $(A.9)$.

Dynamic Responses of Cyclical Components

The observed dynamic responses are computed using the unrestricted VARs, as well as the relations $\ln(Y_t)^c = y_t - \sum_{j=1}^{\infty} E_t\left[z_{t+j}\right]$, $\ln(C_t)^c = c_t - \sum_{j=1}^{\infty} E_t\left[z_{t+j}\right]$, $\ln(I_t)^c = i_t - \sum_{j=1}^{\infty} E_t\left[z_{t+j}\right]$, and the identities $\mathbf{m}_t = \widehat{\mathbf{m}}_t + \theta_{11}\mathbf{p}_t$ and $\mathbf{p}_{t+1} = \widehat{\mathbf{p}}_{t+1} + \theta_{21}\mathbf{p}_t$. As an example, consider the dynamic responses of the cyclical components of the logarithm of output. First, we construct the infinite sum of expected future technology growth using the unrestricted VAR for output: $\sum_{j=1}^{\infty} E_t\left[z_{t+j}\right] = B_{\boldsymbol{y}} \mathbf{x}_{\boldsymbol{y}t}$, where $B_{\boldsymbol{y}} = e_1\left[I_3 - \Gamma_{\boldsymbol{y}}\right]^{-1}\Gamma_{\boldsymbol{y}}$ and I_3 is an identity matrix. Second, we use the responses $(A.10)$, $(A.12)$, and $(A.13)$. We also construct the responses of $\mathbf{x}_{\boldsymbol{y}t}$ using the unrestricted VAR for output:

$$R_{x_y,j} = \Gamma_{\boldsymbol{y}}^j \Lambda_{\boldsymbol{y}} \bar{e}'. \tag{A.22}$$

Finally, the observed dynamic responses are computed as:

$$R_{\ln(Y)^c,j} = R_{\boldsymbol{y},j} - B_{\boldsymbol{y}} R_{x_y,j}. \tag{A.23}$$

Similarly, the predicted dynamic responses are constructed using the restricted VARs and relevant relations and identities. For output, we first construct the infinite sum using the restricted VAR for output: $\sum_{j=1}^{\infty} E_t\left[z_{t+j}\right] = B_{\boldsymbol{y}}^r \mathbf{x}_{\boldsymbol{y}t}$, where $B_{\boldsymbol{y}}^r = e_1\left[I_3 - \Phi_{\boldsymbol{y}}^r\right]^{-1}\Phi_{\boldsymbol{y}}^r$. We then use the responses $(A.15)$, $(A.17)$, and $(A.18)$, as well as the responses of $\mathbf{x}_{\boldsymbol{y}t}$ constructed using the appropriate restricted VAR:

$$R_{x_y,j}^r = \Phi_{\boldsymbol{y}}^j \Lambda_{\boldsymbol{y}}^r \bar{e}'. \tag{A.24}$$

Finally, the predicted dynamic responses are computed as:

$$R_{\ln(Y)^c,j}^r = R_{\boldsymbol{y},j}^r - B_{\boldsymbol{y}}^r R_{x_y,j}^r. \tag{A.25}$$

Data Appendix

This appendix describes the U.S. seasonally adjusted quarterly data covering the 1960:II

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Table 1. Volatility of Growth Rates

	$\sigma_{\Delta \ln(Y)}$	$\sigma_{\Delta \ln(C)}$	$\sigma_{\Delta \ln(I)}$	$\frac{\sigma_{\Delta \ln(C)}}{\sigma_{\Delta \ln(Y)}}$	$\frac{\sigma_{\Delta \ln(I)}}{\sigma_{\Delta \ln(Y)}}$
U.S. Data	0.856	0.512	2.294	0.598	2.679
Basic	0.787	0.488	1.763	0.620	2.241
	(0.000)	(0.229)	(0.000)	(0.290)	(0.000)
Augmented	0.825	0.538	3.010	0.652	3.649
	(0.813)	(0.879)	(0.463)	(0.735)	(0.260)

Note: σ_x denotes the standard deviation of x_t in percentages, where $x_t = \Delta \ln(Y_t)$, $\Delta \ln(C_t)$, and $\Delta \ln(I_t)$. $\Delta \ln(Y_t)$, $\Delta \ln(C_t)$, and $\Delta \ln(I_t)$ are the growth rates of per capita output, per capita consumption, and per capita investment. Numbers in parentheses are p-values associated with a $\chi^2(1)$ statistic of the test that the ratio of predicted to observed volatility is unity. This statistic uses the variance of the ratio, which is computed as $D'\Xi D$ — where D is the vector of numerical derivatives of the ratio with respect to the parameters of the appropriate laws of motion, and Ξ is the covariance matrix of these parameters.

Table 2. Cross-Correlation of Growth Rates

	corr $[\Delta \ln(C_t), \Delta \ln(Y_{t+k})]$						
k	-4	-2	-1	0	1	2	4
U.S. Data	0.087	0.296	0.344	0.764	0.395	0.275	0.128
Basic	0.008	0.061	0.064	0.813	0.090	0.003	0.004
	(0.000)	(0.000)	(0.000)	(0.437)	(0.000)	(0.000)	(0.000)
Augmented	0.099	0.146	0.141	0.747	0.609	0.196	0.018
	(0.970)	(0.609)	(0.489)	(0.870)	(0.045)	(0.768)	(0.746)

	corr $[\Delta \ln(I_t), \Delta \ln(Y_{t+k})]$						
k	-4	-2	-1	0	1	2	4
U.S. Data	0.139	0.173	0.330	0.827	0.318	0.197	0.123
Basic	-0.006	-0.042	0.282	0.928	0.126	0.000	0.001
	(0.000)	(0.000)	(0.005)	(0.000)	(0.000)	(0.000)	(0.000)
Augmented	0.000	0.159	0.585	0.804	0.448	0.130	0.014
	(0.007)	(0.862)	(0.132)	(0.915)	(0.278)	(0.353)	(0.007)

Note: corr$[x_t, \Delta \ln(Y_{t+k})]$ denotes the correlation between x_t and lags k of $\Delta \ln(Y_t)$, where $x_t = \Delta \ln(C_t)$ and $\Delta \ln(I_t)$. $\Delta \ln(Y_t)$, $\Delta \ln(C_t)$, and $\Delta \ln(I_t)$ are the growth rates of per capita output, per capita consumption, and per capita investment. Numbers in parentheses are p-values associated with a $\chi^2(1)$ statistic of the test that the ratio of predicted to observed correlation is unity. This statistic uses the variance of the ratio, which is computed as $D'\Xi D$ — where D is the vector of numerical derivatives of the ratio with respect to the parameters of the appropriate laws of motion, and Ξ is the covariance matrix of these parameters.

Table 4. Cross-Correlation of Cyclical Components

	corr $[\ln(C_t)^c, \ln(Y_{t+k})^c]$						
k	-4	-2	-1	0	1	2	4
U.S. Data	0.787	0.671	0.675	0.679	0.666	0.654	0.638
Basic	-0.285	-0.129	-0.136	-0.101	-0.157	-0.151	-0.117
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Augmented	0.934	0.948	0.960	0.970	0.949	0.929	0.905
	(0.563)	(0.254)	(0.249)	(0.248)	(0.264)	(0.286)	(0.354)
	corr $[\ln(I_t)^c, \ln(Y_{t+k})^c]$						
k	-4	-2	-1	0	1	2	4
U.S. Data	0.118	0.146	0.159	0.173	0.179	0.185	0.366
Basic	0.169	0.037	0.042	0.023	0.070	0.067	0.558
	(0.913)	(0.690)	(0.679)	(0.624)	(0.685)	(0.652)	(0.745)
Augmented	0.189	0.205	0.215	0.226	0.231	0.237	0.248
	(0.983)	(0.989)	(0.990)	(0.991)	(0.992)	(0.992)	(0.990)

Note: corr$[x_t,\ln(Y_{t+k})^c]$ denotes the correlation between x_t and lags k of $\ln(Y_t)^c$, where x_t=$\ln(C_t)^c$ and $\ln(I_t)^c$. $\ln(Y_t)^c$, $\ln(C_t)^c$, and $\ln(I_t)^c$ are the cyclical components of the logarithms of per capita output, per capita consumption, and per capita investment. Numbers in parentheses are p-values associated with a $\chi^2(1)$ statistic of the test that the ratio of predicted to observed correlation is unity. This statistic uses the variance of the ratio, which is computed as $D'\Xi D$ — where D is the vector of numerical derivatives of the ratio with respect to the parameters of the appropriate laws of motion, and Ξ is the covariance matrix of these parameters.

Figure 1. Dynamic Responses of Growth Rates: Basic

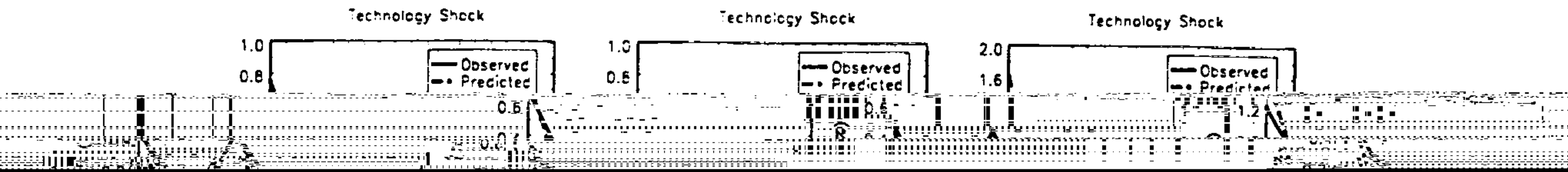
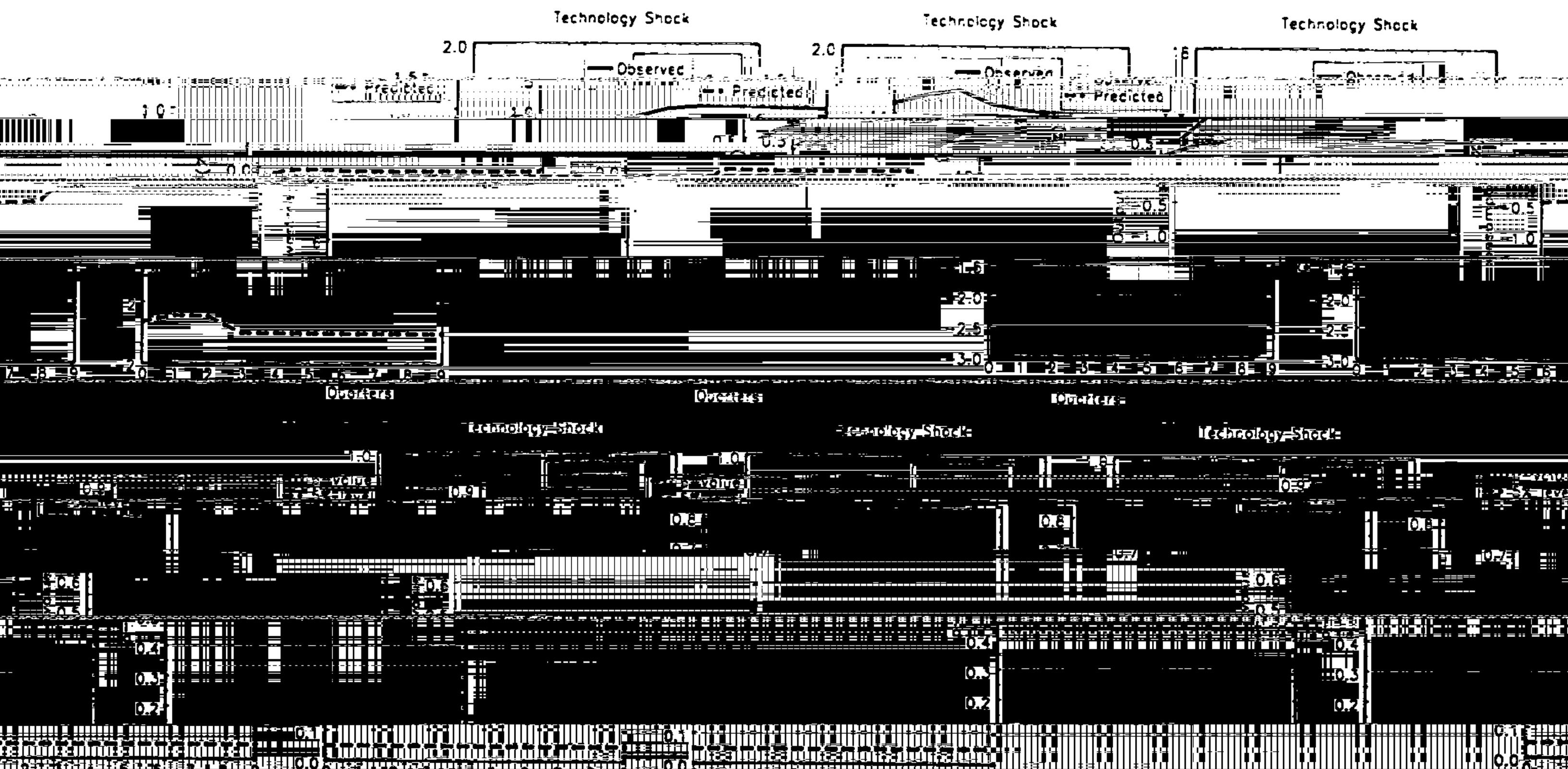
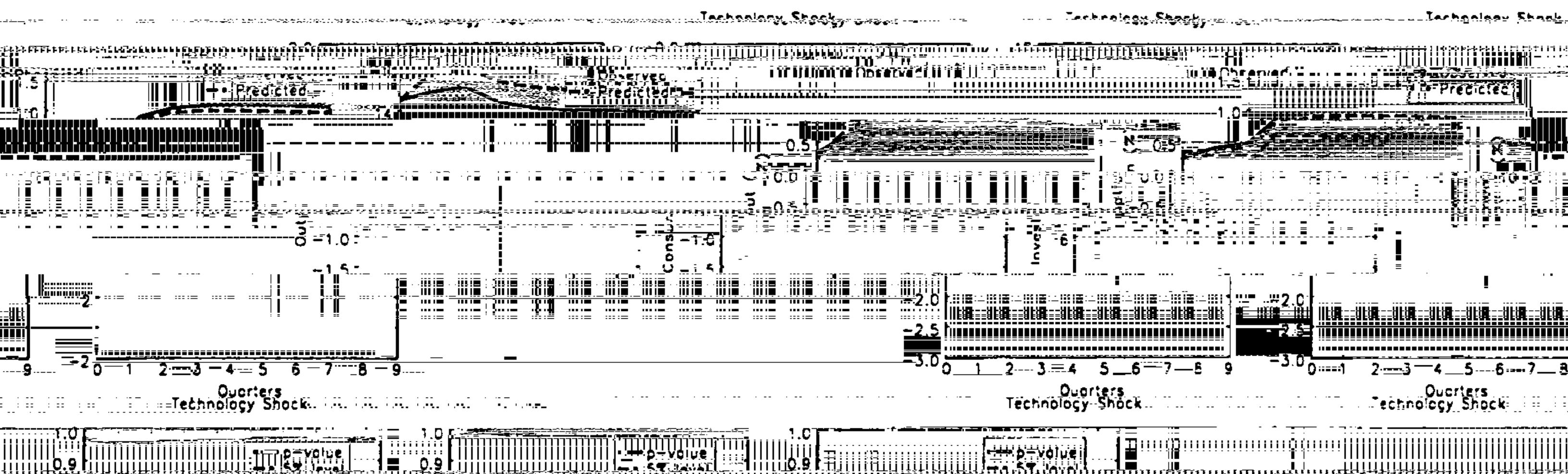


Figure 2. Dynamic Responses of Growth Rates: Augmented

Figure 2. Dynamic Responses of Cyclical Components: Basic





Technology Shock
Predicted
Quarters
p-value