DISCUSSION PAPERS IN ECONOMICS

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Does Evolution Solve the Hold-up Problem?

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1 Introduction

[illegible]

2 Investment and Bargaining

$D_A \vdash \{V\ I() - x; V\ I()\}$ ... B ... A ... A ... D ... B ... A ... $I; y\ x()$ ... A ... $y\ D \to D$ ...

... A ... $I \vdash I^*\ I$

3 Evolution

A *B* *population* *N* $t \in \{1; ; ...\}$ *A* *B* *beliefs* *A* *B* *B* *A* *I* *B* *x*

Assumption 1 *(i) The pie division is small: V I > –. (ii) The population is large: V I** *adaptation*

random mutation.

Θ *basin of attraction* μ B $\mu()$ μ' μ' μ μ' *single mutation neighborhood* μ $\mu' \in M$ $\mu()$ μ' μ X *mutation connected set* μ_1 $\mu_n \subset X$ $\mu_2, \ldots, \mu_{n-1}()$ k $; \ldots; n;$ M $\mu() \cap BX\mu$

$$x^L = \max\{x \in D_B(I^*) | (V(I^*) - x)\frac{N-1}{N} - I^* \geq V(I) - I\}.$$

$$x^M = \max\{x \in D_B(I^*) | V(I^*) - x - I^* > V(I) - I\}.$$

$x^M \leq x^L \leq x^M.$

$$\begin{aligned} V^* - x^M (N-1)/N - I^* &= V(I^*) - x^M - I^* - (V(I^*) - x^M)/N \\ &\quad - (N-1)/N \\ &> V(I^*) - x^M - I^* - x^M/N \\ &\geq V(I^*) - x^M - I^* \\ &\geq V(I) - I; \end{aligned}$$

$(N-1)/N$ x^L B A

Proposition 3 *Let agents bargain according to the Nash demand game. The outcome ‰ is locally stable if and only if* ‰ $= \{I^*; V(I^*) - x; x\}$, *where* $x \leq x^L$.

Theorem 1 *An equilibrium μ is stochastically stable if and only if no other equilibrium has lower stochastic potential.*

$r\ x() > r\ x()$ $x > x^L$

$x^M > x^{NBS}$

$r\ x^L() \geq r\ x^L - -();$

$$- \longrightarrow\ - \longrightarrow :::\longrightarrow x^L - - \longrightarrow x^L;$$

$r\ x^L() \geq r\ x^L - -()$

$$x^L \longrightarrow - \longrightarrow\ - \longrightarrow ::: \longrightarrow x^L - -:$$

[illegible]

Appendix: Proofs

[illegible]

Lemma 1 *Let* $z_1 < z_2 ::: < z$ *be demands in* $D\,I()$ *for some* $I \in \Psi$. *Assume that the set of demands following* I *for agents in the relevant population is* $\{z_l\}_l$

[illegible] □

Lemma 4 *Let* μ' *(* $\mu'(\cdot) = \{l'; y'; x'\}$ *) be an equilibrium. If* $l \neq l'$ *and* $y - l \geq y' - l'$*, then the population can get from* μ' *to an equilibrium* μ *with* $\mu(\cdot) = \{l; y; x\}$ *through a sequence of single mutation transitions.*

[illegible] □

[illegible]

[(st3 1 Tn)411

$I; V \; I$

$\{ I^*; y; x \}$ $x \leq x^L$

$V \; I^* - x^L \; N - I = N - I^* \leq V \; I^* - x \; N - I = N - I^*$

[illegible] A [illegible] $(I; V - -; -)$ [illegible] B [illegible] $x^L \in \{x^M; x^M - -\}$:

Lemma 5 *The number of mutations required to get from an equilibrium with outcome $(I^*; y; x)$ with $x \leq x^L$ to an equilibrium with outcome $(I; V - -; -)$ is $r(x) = \min\{r | r > N(1 - \frac{\hat{V} - \delta - \hat{I} + I^*}{V^* -})\}$.*

[illegible] r [illegible] B [illegible]

$$\frac{N - r}{N}(V^* - x) - I^* < V - I - -;$$

[illegible] □

Lemma 6

(i) If µ is an equilibrium with outcome $(I^*; y; x)$ and $x < \min\{x^M; x^{NBS}\}$;
the easiest transition away from ‰ µ [illegible]

$x > x^L$

x x^L x^M $x^M < x^{NBS}$;

$r(x) \geq r(x)$ $V^* - x^M - - \leq V - - - I$ □

x $x^M > x^{NBS}$

$I; V - -; -)$ $(I^*; y$ $-; x - -)$

$(I^*; y; x)$ y $V^* - x)$ $(I^*; y'; x')$

Lemma 7 *From an outcome $(I^*; y; x)$ the easiest transition in which investment is at all times e–cient, but which ends with difierent demands, is to an outcome $(I^*; y'; x')$ where x $x - -; x$ $-; -;$ or $V^* - -$.*

□

$-;$

$-$

Lemma 8

(i) Moving from x to $x - -$ takes N $\left(- \frac{-\delta}{}\right)$ mutations to pop A.

(ii) Moving from x to x $-$ takes N $\left(- \frac{V^* - -\delta}{V^* -}\right)$ mutations to pop B.

(iii) Moving from x to $-$ takes N $\left(\frac{V^* -}{V^* - \delta}\right)$ mutations to pop B.

(iv) Moving from x to $V^* - -$ takes N $\left(\frac{}{V^* - \delta}\right)$ mutations to pop A.

N □

Lemma 9

(i) If $- < x < V^* - -$, then moving from x to $x - -$ takes fewer mutations than moving from x to $-$, and moving from x to x $-$ takes fewer mutations than moving from x to $V^* - -$.

(ii) If x ⌐ *– then moving from x to*

... $x^L > x^{NBS}$ ... Θ^1 ... Γ^1 ... $x < x^{NBS}$ ... $\mu, \mu'_{+\delta}$ ... $x^{NBS} < x \le x^L$ ... $\mu, \mu'_{-\delta}$ ... $r(x) < r(x-)$ ... $x < x^{NBS}$ ... $r(x) > r(x-)$ ... $x > x^{NBS}$. □

... A ... B ... B ... l ... A ... y ...

Lemma 11 *Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome, $(l^H, V^H - x^{\max}(l^H), x^{\max}(l^H))$, is a subset of the unique locally stable set.*

... $\exists \mu^H$ ... $\mu^H \in T(\mu)$ ... μ ... $\%(\mu^H) = \{(l^H, V^H - x^{\max}(l^H), x^{\max}(l^H))\}$ ... μ ... $\%(\mu) = \{(l, V(l) - x, x)\}$ ... A ... $x^{\max}(l')$ ... $l' \neq l$ ... $x^{\max}(l)$ ... l ... B ... $x^{\max}(l)$ ... B ... A ... $(l^H,$ ...

□

Lemma 12 *Let surplus be divided by the ultimatum game. Agents in population A receive a payoff of at least $V^H - l^H - x^{\max}(l^H)$ in every equilibrium.*

... l^H ... A ... l^H. □

Lemma 13 *Let surplus be divided by the 'ultimatum' game. If $V(l) - l - x \ge V^H - l^H - x^{\max}(l^H)$, then there exists an equilibrium μ such that $\mu \in \Theta^L$ and $\%(\mu) = (l, V(l) - x, x)$.*

6 References

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