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Does Evolution Solve the Hold-up Problem?

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1 Introduction

pp pp rrl pb _

pg pl pl r^kr

ll r f r p p l r k r g p p g g m r b l p
 p b m l ll r l p k r p p b f b r g p p g
 g m b p q a l r m b b r k r p p b r p
 b p j a p b k r g p p g m b b p r l
 r p g p f r r b b k b f ll p g a p
 • \W b f b r g p p g g m b m l l g m r f a l r r'
 • \W b f m l b r l p p b p g m r f p'

l p p p r p g m l p l b m l r b l p
p p l f r g m r f l r p f r m r g p p g
r m p b l p b r r f g m r f
p m p b l p l r m l r g p p g g m
b p g g m r f a l r m p m l p l m m g m p
b b b p r p m r r b r l m b r k p g
r p r r l r p g m r f p b g m r b r k p g
l m l f b l r l m b p r b l p r
p m r b m g r b p b r n g r p r b l p p r ll
b b l r l p p p l b p r b l m k r ll p
p m p b l b m b ll l m p b r
m p l W n g b p p m p l l
l b g b r m l f r g p p g g m b r l
g g b g p r l r p l m g b r f b p b r p p
p p p p g m r f p b W l p p l
k m p m l l p a b m p b
b r b p b p p p g m r f p r p m b
l b l l n g r f b p g p r l f r b
r g p p g g m r r p r m p l p r
p l f b b l r l m n g b r p p r r g p p g g m
p m p p m p
b l r p l l p r b r l p g p r p l
p W r p n g b p f r r p p b r p b b
p r l r b r f b r l b p r p k
b b r n g r p r l b ll p m r g
f r ll b r r p l l r p b b p m p p r
k p b r b p r r p p p f r l a l r m
r b m m k b f r r p p r g m p b b r n g
r p r b l r a r b r p b r p l b r b p
b r n g r p r b l p r p b p a l r m b b m k
b r l l f p m p r p k f r b p r p m

2 Investment and Bargaining

$\mathcal{W} = \{ \omega \in \mathbb{R}^N : \omega \geq 0, \sum_{i=1}^N \omega_i = 1 \}$

$\Psi = \{ \psi = (\psi_0, \psi_1, \dots, \psi_N) \in \mathbb{R}^N : \psi_i \geq 0, \sum_{i=0}^N \psi_i = 1 \}$

$V = \{ v \in \mathbb{R}^N : v_i \geq 0, \sum_{i=1}^N v_i = 1 \}$

$\mathcal{W} \times \Psi \times V$

$\mathcal{S} \text{ gm } \mathcal{R}^f \text{ p m } r \text{ ll } p \text{ m } p \text{ b } b \text{ m } p \text{ gm}$
 $p \text{ l } m \text{ l } p \text{ m } p \text{ b } l \text{ m } m \text{ gm } b \text{ p } l \text{ b}$
 $p \text{ l } p \text{ r } \text{ ll } r \text{ p } b \text{ p } l \text{ b } r \text{ r } p \text{ f } l \text{ p } r$
 $l \text{ p } \text{ (} \text{ gm } \mathcal{R}^f \text{ p}$

3 Evolution

$\text{ } \text{ l } p \text{ r } p \text{ l } r \text{ f } p \text{ f } r \text{ m } p \text{ r } p \text{ g } b \text{ m } b \text{ l } p \text{ f}$
 $\text{ } b \text{ b } r \text{ g } r \text{ r } p \text{ l } p \text{ r } p \text{ r } b \text{ m } r \text{ p } m$
 $\text{ } p \text{ f } b \text{ b } r \text{ g } p \text{ r } m \text{ r } r$
 $\text{ } b \text{ f } b \text{ l } p \text{ p } g \text{ m } p \text{ r } r \text{ p } p \text{ g}$
 $\text{ } \text{) } p \text{ r } l \text{ b } p \text{ l } \text{ k } \text{) } p \text{ p } g \text{ l } \text{) } b \text{ p } p$
 $\text{ } p \text{ f } r \text{ p } g \text{ m } l \text{ k } \text{) } p \text{ m } l \text{ p } l \text{) } p \text{ l}$
 $\text{ } b \text{ r } f \text{ r } m \text{ r } k \text{ b } r \text{ r}$

$r \text{ b } l \text{ r } r \text{ l } A \text{ p } B \text{ l } b \text{ r } \text{ population } \text{ f } N \text{ b}$
 $r \text{ t} \in \{l; m\} \text{ r } l \text{ m } p \text{ p } \text{ f } g \text{ p } p \text{ l } p \text{ A } p$
 $B \text{ p } p \text{ l } b \text{ p } m \text{ p } m \text{ r } g \text{ p } p \text{ g } g \text{ m } b \text{ f } r \text{ g } k$
 $b \text{ m } g \text{ p } l \text{ b } l \text{ beliefs } b \text{ r } p \text{ p } \text{ f } p \text{ g}$
 $p \text{ l } p \text{ r } r \text{ b } p \text{ r } g \text{ r } l \text{ r } b \text{ r } p \text{ ll } g \text{ p}$
 $b \text{ r } l \text{ f } \text{ " } \text{ l } \text{) } p \text{ l } r \text{ A } l \text{ f } p \text{ r } p \text{ g } l \text{ r } B \text{ m } p$
 $p \text{ l } \frac{3}{4} \text{ l } \text{) } p \text{ l } r \text{ B } l \text{ f } l \text{ r } A \text{ m } p \text{ b } \text{ "}$
 $p \text{ } \frac{3}{4} \text{ r } r \text{) } l \text{ r } p \text{ p } b \text{ f } l \text{ m } p \text{ p } p \text{ b}$
 $r \text{ p } p \text{ g } p \text{ b } p \text{ m } p \text{ l } \text{ f } r \text{ l } r \text{ p } g \text{ b}$
 $l \text{ m } m \text{ g } m \text{ } \frac{3}{4} \text{ l } p \text{ p } l \text{ r } B \text{ m } p \text{ x}$
 $\text{ } \text{ W } \text{ f } p \text{ l } \text{ [} b \text{ p } l \text{ m } p$

Assumption 1 (i) The pie division is small: $V \text{ l } \text{) } > -$. (ii) The population is large: $V \text{ l } \text{) }^*$

♭ g p ♭ μ b r p
r l f p r p f r p p l p W b b p z μ)
l f p r g l p r p adaptation b
r p p r p p p random mutation. p r p b f l
l p g r r b g p b p b p f r p ll p g
b l f p r g b ll p p g r p p p g g p
r z μ) b l f p b r p l f f ll p g
p p p r b p μ r p b p g) p b r

$f: X \rightarrow X$ continuous map, $\mu \in M(X)$ probability measure, $B(\mu)$ basin of attraction, μ' single mutation, neighborhood of μ , $M(\mu)$ mutation connected set, μ_1, \dots, μ_{n-1}

$\{x \in D_B \mid V(x) \geq V(I^*)\}$

$$x^L \in \{x \in D_B \mid V(x) \geq V(I^*)\}$$

$\{x \in D_B \mid V(x) \geq V(I^*)\}$

$$x^M \in \{x \in D_B \mid V(x) \geq V(I^*)\}$$

$x^M \leq x^L \leq x^M$

$$\begin{aligned}
 V^* - x^M &\geq N - I^* - V(I^*) - x^M - I^* - V(I^*) - x^M = N \\
 &\quad - N - I^* = N \\
 &> V(I^*) - x^M - I^* - x^M = N \\
 &\geq V(I^*) - x^M - I^* \\
 &\geq V(I^*) - I^*
 \end{aligned}$$

$x^L \in \{x \in D_B \mid V(x) \geq V(I^*)\}$

Proposition 3 Let agents bargain according to the Nash demand game. The outcome x_0 is locally stable if and only if $x_0 \in \{I^*; V(I^*) - x; x\}$, where $x \leq x^L$.

$\{x \in D_B \mid V(x) \geq V(I^*)\}$

$\mu\{tree, \Gamma$ \parallel p \langle f g p G \rangle b b \langle f r m r r μ \rangle μ \langle μ \rangle
 μ \langle r Γ \rangle b m \langle f b r \rangle p \langle \parallel b g p b r \rangle p \parallel
 b $stochastic$ $potential$ \langle p g l r m μ \rangle b m p m m r \rangle p r \parallel
 μ r b f b f g b b r p g l r m b \parallel l
 r p g l b r m \rangle

Theorem 1 *An equilibrium μ is stochastically stable if and only if no other equilibrium has lower stochastic potential.*

p \langle f b p l p b r \langle f f r \rangle p \rangle b p
 r r b r m b m l r b p b r \rangle p p
 p r p p p g l r r g r l p m p p b p
 p r p g m p m r p r p g r g b r p l
 l b \langle f r m p g l r m p m r \rangle p g l r m b p
 l \parallel l b r g b p \langle f p m p r p \rangle p p
 m m r p b p b l \parallel l p b m m p p r p
 p r r b l \parallel l r r p l \parallel l
 m \rangle r
 s p \parallel l \parallel l m p r b m p r g p p g b b
 r r b l r l^* p b y l V l^* \rangle $-x$ b m p \langle \parallel
 b r r B m p x b r r p b b r p p
 p l \parallel l m m r r b r m g r
 r p p r g b b p m p m p p b p l l p
 b p l p g l \rangle p r b \langle f $x < x^{NBS}$
 V l^* \rangle $;$ b p b r p p b m x $-$ p r l \langle
 $x > x^{NBS}$ \rangle b r p p x $-$ \rangle p b r p b
 r p p r x g p b r m g p p r r p p r g b b
 b l p b r g b p p p m p b p l
 b g p l l r g r p \langle l p B p r b r m p
 b p p b p m p l r l p
 A b p b m l V l \rangle $-$ $-$ \rangle b b b m r p p
 m \langle f b b p m g \langle \parallel g r r \rangle r \langle f b p
 b l p \parallel m f r p p b p p m \langle f r b b
 g p \langle g l m p r p p b r p \langle f b b p g r \rangle
 g b l p l^* V l^* \rangle $-$ $-$ \rangle r x \rangle p b r p
 \langle f b r p p r m l^* V l^* \rangle $-x$ x \rangle l V l \rangle $-$ $-$ \rangle s p

$$\begin{array}{c}
\begin{array}{c}
p \ b \ b \ r \ X \rangle > r \ X \rangle \ b \ p \ r \ X > X^L \\
p \ r \ p \ p \ p \ p \ r \ p \ r \ b \ X^M > X^{NBS} \ p \ \rangle \ \parallel \ r \\
b \ p \ l \ \{ \ b \ b \ p \ p \ g \ p \ p \ p \ g \ l \ \} \ b \ r \ \{ \\
r \ X^L \rangle \geq r \ X^L - \rangle \ b \ p \ p \ p \ p \ r \ p \ r \ g \ p \ \}
\end{array} \\
\longrightarrow \longrightarrow \dots \longrightarrow X^L \longrightarrow X^L; \\
b \ l \ \{ \ r \ X^L \rangle \geq r \ X^L - \rangle \ b \ p \ p \ p \ p \ r \ p \ r \ g \ p \\
X^L \longrightarrow \longrightarrow \dots \longrightarrow X^L \longrightarrow
\end{array}$$

p p p b p b r p p p p p l l r p⁸
l s p b r g p p g p r p g b r l f b
l p p g p p /^H b b V /^H - /^H

$\mathbb{W} \setminus \{z_i\} \cap \mathbb{Q} \neq \emptyset$

Appendix: Proofs

$\mathbb{W} \setminus \{z_i\} \cap \mathbb{Q} \neq \emptyset$

Lemma 1 Let $z_1 < z_2 < \dots < z_n$ be demands in $D \setminus \{z_i\}$ for some $i \in \Psi$. Assume that the set of demands following i for agents in the relevant population is $\{z_i\}_i$

p p p l p l ll g p p b r l p b p k
 l b p p V l z p p p g p l ll p p k
 p p z p b p p b r p p r r
 b b p r b p p b Q p r pg sp M p N
 p r l y r p p x b r l r p
 f ll pg l sp b p p r r b r r g pg l p
 ll b r f r b l p p g l r p Q pg p \square
 r b p r r r p
 \backslash W r b p r r r p
 r f f r p p b Q p pg p sp pg
 p b pg b l f $\%$

(ii) If x_{1-} - then moving from x to

$x^{NBS} < x \leq x^L$
 $\mu; \mu'_{\neq \delta}$
 $x < x^{NBS}$
 $\mu; \mu'_{-\delta}$
 $x > x^{NBS}$
 \square

Detailed mathematical proof for Lemma 11 involving various variables and set definitions.

Lemma 11 *Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome, $\{I^H; V^H - x^{\max}, I^H\}$, is a subset of the unique locally stable set.*

Detailed mathematical proof for Lemma 11 involving various variables and set definitions.

Lemma 12 *Let surplus be divided by the ultimatum game. Agents in population A receive a payoff of at least $V^H - I^H - x^{\max}$ in every equilibrium.*

Detailed mathematical proof for Lemma 12 involving various variables and set definitions.

Lemma 13 *Let surplus be divided by the 'ultimatum' game. If $V - I - x \geq V^H - I^H - x^{\max}$, then there exists an equilibrium μ such that $\mu \in \Theta^L$ and $\mu \in \{I; V - I - x; X\}$.*

r f m m f r m m l p □
 b f r f m g b m A b g r b p
 b g p b b l a l r m b p b r r g l r p b b b
 m r r p b l m m r m p b r b p
 g l r p r r p b l r b f b r l m m
 r f f r p r m m l l p l k b b b
 p g l ll l b b s m l p l b p m g l b
 b ll l □

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