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Economic Principles of Space Traffic Control

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Abstract

Open access to Earth's orbits presents a unique regulatory challenge. Technical solutions to space traf c control tend to emphasize launch restrictions or public funding of debris removal technology development and use, but often ignore that current and prospective orbit users dissipate rents under open access. In this paper, I derive economic principles governing the choice of space traf c control policies. I show that policies which target satellite ownership, such as satellite taxes or permits, achieve greater expected social welfare than policies which target satellite launches, such as launch taxes or permits. Price or quantity policies can achieve equal expected social welfare due to the symmetry of uncertainty between regulators and rms. I also show that active debris removal can reduce the risk of runaway debris growth no matter how it is nanced, but can only reduce the risk of satellite-destroying collisions if satellite owners pay for it or if competition from removal-induced entry reduces the returns to satellite ownership. My results show that attempts to control orbital debris growth and collision risk through launch fees, debris removal subsidies, or purely technical solutions may be ineffective or back re.

JEL codes: Q28, Q54, Q55

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1 Introduction

Open access to common-pool resources tends to cause resource overuse or stock collapse. Open access orbit use has led to the accumulation of orbital debris, from nonoperational satellites to nuts, bolts, and propellant fuel particulates. Collisions between orbiting bodies can shatter satellites into thousands of dangerous high-velocity fragments, some of which may be too small to track. Runaway debris growth, known as Kessler Syndrome, threatens to render high-value orbits unusable for decades or centuries. As technology makes satellites cheaper to launch and more reliable, rms are planning to launch thousands of satellites into already-congested orbits. The need for policies to manage orbital congestion is more pressing than ever. Unfortunately, engineers, economists, and policymakers know little about how space traf c should be managed and debris removal technologies should be employed. In this paper I answer two fundamental questions of space traf c control. First, what do optimal space traf c control policies look like? Second, how should active debris removal be nanced? The key insights of my paper are that space traf c control policies should target satellites in orbit rather than satellite launches, and that satellite owners must pay for debris removal for it to reduce equilibrium collision risk.

I derive economic principles of space traf c control policy in the rst dynamic model of satellite launch and ownership with physical uncertainty over collisions and positive feedbacks in debris growth. I highlight the key policy design constraints imposed by open access and show how the use of active debris removal technologies will affect equilibrium collision risk and debris growth. I show that despite uncertainty over the risk of catastrophic collisions, the traditional “prices vs. quantities” question is moot. Price or quantity policies can achieve rst-best outcomes because both regulators and rms are equally uncertain about the collision risk. The key design issue is whether the regulator's policy targets satellites in orbit (for example, a satellite tax) or the act of launching satellites (for example, a launch tax). In the setting I study, regulating satellites in orbit achieves higher expected social welfare than regulating the act of launching satellites. Regulating satellite launches instead of satellites in orbit creates and collision risk (Liou and Johnson, 2008, 2009b; Bradley and Wein, 2009; Ansdell, 2010; Schaub et al., 2015; Macauley, 2015 controls, and Proposition 6, that satellite owners must pay for debris removal if the technology is to reduce equilibrium collision risk. I show proofs of these and a few other economically important results in the main text (the rest are in the Appendix, section 8). Finally, I conclude in section 4 with discussion of the results and thoughts on the future of commercial orbit use.

2 Essentials of Orbit Use

In this section I discuss the history and current status of space traf c control policies. Readers interested in going directly to the modeling approach may skip to section 2.2. Readers interested in learning more of the institutional details of orbit use may go to the Appendix, section 5.

2.1 De ning “space traf c control”

One of the central challenges of space traf c control is how to de ne “space traf c control”. Nicholas Johnson, a scientist at NASA, has proposed an aim of space traf c control: “...the goal of space traf c management is to minimize the potential for (radio frequency) or physical interference at any time” Johnson (2004). The radio frequency interference problem is relatively tractable and being handled by existing institutions (Jones et al., 2010). The physical interference problem, essentially collision avoidance, is more dif cult from technical and legal perspectives. In GEO, space traf c control is “position control”: since satellites in GEO have very low speeds relative to each other, traf c control is as simple as spacing satellites far enough apart that they are unlikely to collide or cause radio frequency interference. In current regulatory regime, the International Telecommunications Union assigns frequency blocks and geostationary “slots” to national authorities. These authorities are then free to assign their frequencies and slots to entities within their jurisdiction as they see t, and are also responsible for enforcing responsible spectrum use. In the United States, this is handled by the[3] FCC.

Space traf c control in LEO is harder than in GEO. Satellites in LEO are constantly in motion with respect to each other and have little or no control over their trajectories. Notions like “keep-out zones” are impractical since satellites may only occasionally or accidentally pass through them, and concepts like “rules of the road” raise the question of how a road is to be de ned in LEO. Figure 1 shows the orbits of 56 cataloged satellites with mean altitudes of 700-710 kilometers, and makes the inaptness of road, sea, and air analogies clear. The growth in LEO use has motivated calls for broader notions of space traf c control which encompass non-GEO regimes. There are currently no international regulatory agencies which

[3]Readers interested in more detail about the history and institutions of space traf c control are referred to Johnson (2004); Jones et al. (2010). Technical proposals for mass removal are discussed in Klinkrad and Johnson (2009), Weeden (2010) discusses the legal challenges, and Tkatchova (2018) examines the potential for markets in debris removal.

coordinate launches and satellite placements to manage debris growth and collision risk; the extent of management policies currently is a patchwork of national regulations and non-binding international guidelines. Table 1 shows the breakdown of currently-operational satellites by location of launch site to emphasize the international dimension of the problem. Figure 2 shows the growth in orbit use from active satellites and debris, as well as the increase in competition to provide commercial launch services.

[Figure 1 about here.]

[Table 1 about here.]

For this paper, I de ne space traf c control as policies or technologies intended to manage the probability of collisions between active satellites and other bodies. This de nition encompasses satellite path as well as debris growth management. Any space traf c control policy, including command-and-control regulations, can be characterized as a price or quantity control, such as a tax or a quota. If the effect of a policy is to raise the cost or limit the availability of satellite launch, I label it as a “ ow” control. If the effect is to raise the cost of operating a satellite or constrain the allowed number of satellites in orbit, I label it as a “stock” control. The existing patchwork of policies includes both ow controls intended to manage launch capacity and prevent launches from interfering with air traf c, and stock controls intended to manage spectrum congestion. While most existing literature on space traf c control focuses on controlling the trajectories of objects in orbit, I focus on controlling the number of objects in orbit. Brief consideration will show that the former implies the latter. I treat debris removal separately because the technology is not yet commercially available, so analysis of a world without debris removal is more immediately relevant to policy design.

[Figure 2 about here.]

2.2 A simple model of orbital mechanics

In this section I describe the laws of motion for orbital stocks, the type of uncertainty most relevant to the economics of managing collision risk and debris growth (symmetric physical uncertainty), and the functional forms I use for simulations. Following analytical debris modeling studies such as Rossi et al. (1998) and Bradley and Wein (2009), I consider the evolution of orbital stocks in an arbitrary spherical shell around the Earth, referred to as the “shell of interest”. More detailed physical models of Earth orbit use multiple shells. I ignore such features in this paper for tractability. I consider two types of ctitious agents: a social planner who launches and owns all satellites in orbit to motivate optimal satellite launch and debris removal plans, and a global regulator who manages all satellites launched or in orbit to motivate policy choice.

Let S_t

of $G(\cdot)$. The expected value at the end of period t of a function $f(\ell_{t+1})$ is

$$E_t[f(\ell_{t+1})] = \int_0^1 f(\ell_{t+1}) f(\ell_{t+1} | S_{t+1}, D_{t+1}) d\ell_{t+1}.$$

I also assume that the distribution of the collision rate is "increasing" in the number of satellites and amount of debris, in the sense that an increase in either satellites or debris results

Economically, the expected collision risk can be thought of as a matching function which matches active satellites to debris and other active satellites. The form in equation 3 implies that matching between active satellites and debris or other active satellites exhibits "thick market effects": one more active satellite or unit of debris increases the ease with which all active satellites are matched with other orbital bodies. The economic intuition of the expected collision risk function is discussed in more detail in Rao and Rondina (2018).

2.2.2 Kessler Syndrome

Kessler Syndrome is a central concern in orbit use management. If open access can prevent Kessler Syndrome, regulating orbit use is not as important from an environmental perspective. Even though orbit use will be inefficient it will not cause irreversible environmental damage. On the other hand, if open access can cause Kessler Syndrome, orbit use management is more urgent.

In this section I formally define Kessler Syndrome and establish some properties of the debris threshold beyond which it occurs. Open access debris levels are increasing in the excess return on a satellite while the Kessler threshold is constant, implying that sustained increases in the return on a satellite can cause Kessler Syndrome under open access. Though the Kessler threshold is defined purely in terms of the system's physics, the occurrence of Kessler Syndrome depends critically on the economics of orbit use.

Assumption 2. (Debris growth) The growth in new fragments due to debris is larger than the decay rate for all levels of the debris stock greater than some level $\bar{D} > 0$,

$$\bar{D} : G_D(0, D, \ell) > d \ \forall D > \bar{D} \ \forall \ell.$$

Due to assumption 2 and $G(S, D, \ell)$ being increasing in all arguments, there is a unique threshold $D^k \geq \bar{D}$ above which Kessler Syndrome occurs. Past this threshold, the number of new fragments created by collisions between debris exceeds the amount which decays in a single period. For regimes where this condition doesn't hold at any level of debris, Kessler

Without active debris removal technologies, Kessler Syndrome is an absorbing state. Once and the equilibrium collision risk is

$$E_t[\ell_{t+1}] = r_s - r. \tag{9}$$

The fleet planner maximizes the expected net present value of the entire fleet. Their problem is

$$W(S_t, D_t, \ell_t) = \max_{X_t \geq 0} \{ pS_t - FX_t + \beta E_t[W(S_{t+1}, D_{t+1}, \ell_{t+1})] \} \tag{10}$$

$$\text{s.t. } S_{t+1} = S_t(1 - \ell_t) + X_t \tag{11}$$

$$D_{t+1} = D_t(1 - d) + G(S_t, D_t, \ell_t) + mX_t. \tag{12}$$

The planner launches so that the loss rate is equated to the rate of excess return net of the marginal external cost (ξ_{t+1}), that is,

$$E_t[\ell_{t+1}] = r_s - r - \frac{E_t[\xi(S_{t+1}, D_{t+1})]}{F}, \tag{13}$$

where $E_t[\xi(S_{t+1}, D_{t+1})]$ is the marginal external cost of a satellite launch. For the results in

Earth's magnetic field for propulsion and deploy nets, harpoons, or tethers (for example, Pearson, Carroll, and Levin (2010)) to either deorbit debris or recycle the materials for in-space manufacturing. Ground-based lasers are another candidate technology to deorbit debris.

I assume no new satellites are required to implement removal, which can be interpreted in two ways: that the removal technology is ground-based; or that the satellites required are already in orbit and can never be destroyed or lost. Including the requirement that new satellites be used for removal complicates the model in interesting and relevant ways that are beyond my scope here. I also assume that only satellite owners can purchase debris removal.

With the ability to remove debris from orbit, satellite owners can remove clearly-dangerous pieces of debris before they impact their satellites. The remaining collisions will be caused by errors in debris risk assessments, satellite trajectory forecasts, and collisions which were deemed too costly to avoid. To reflect this in the model, I adjust the timing of when ℓ_t is revealed when debris removal technologies are present. Satellite owners purchase R_t total units of removal before ℓ_t is revealed, with the aim of changing the distribution of ℓ_t until the marginal private benefit of removal equals the marginal private cost. After removal has been purchased, ℓ_t is drawn from a distribution conditioned on S_t and $D_t - R_t$ (instead of just S_t and D_t) and revealed to all satellite owners and prospective launchers. The launchers then decide whether or not to launch.[6]

With debris removal before collisions, the laws of motion and distribution of the collision rate become

$$S_{t+1} = S_t(1 - \ell_t) + X_t \tag{14}$$

$$D_{t+1} = (D_t - R_t)(1 - d) + G(S_t, D_t - R_t, \ell_t) + mX_t \tag{15}$$

$$\ell_t \sim f(\ell_t | S_t, D_t - R_t). \tag{16}$$

Expectations before removal in t are indicated by $\tilde{E}_t[\cdot]$ and treat ℓ_t as a random variable, while expectations after removal in t are indicated by $E_t[\cdot]$ and treat ℓ_t as known. The expected collision risk before removal is effected is

$$\tilde{E}_t[\ell_t] = \int_0^1 \ell_t f(\ell_t | S_t, D_t - R_t) d\ell_t. \tag{17}$$

Potential launchers in t have the same expectations as before: they are aware of ℓ_t and treat

[6] In reality, the timing of satellite launches and debris removals will not be this clearly separated. However, potential launchers will be able to anticipate satellite owners' debris removal demands, and where possible structure their launches to take advantage of these efforts.

ℓ_{t+1} as uncertain. Formally,

$$E_t[\ell_{t+1}] = \int_0^{\infty} \ell_{t+1} f(\ell_{t+1}|S_{t+1}),$$

which are in principle observable by all actors. Unlike the regulatory problems considered in Weitzman (1974) and Newell and Pizer (2003), the firm has no additional information about the motion of orbital bodies for the regulator to harness through instrument design.

The distinction between stock and flow controls is relevant to a broad class of economic management problems. To encourage renewable energy generation, a regulator may weigh investment (stock) vs production (flow) tax credits (Aldy, Gerarden, and Sweeney, 2018). To manage public infrastructure a regulator may weigh investment in damage abatement (flow) vs quality restoration (stock)(Keohane, Van Roy, and Zeckhauser, 2007).[8]

In the absence of informational or administrative constraints on the regulator, the preferred instrument is that which most directly targets the externality-generating activity (Sandmo, 1978). In the renewable energy case, production tax credits can encourage renewable energy generation more effectively than investment tax credits.[9] In orbit, stock controls dominate flow controls because the collision risk externality is driven by the number of objects in orbit rather than the number of objects launched in a period.

Stock and flow controls can often be made equivalent in the sense that one can be capitalized or annuitized to the same present value cost as the other. However, they have different effects on the incentive to launch or own a satellite. Imposing a fee at launch increases the cost of entering the orbital commons, penalizing entrants while increasing the rents accruing to incumbents in orbit. Imposing a recurring fee while the satellite is in orbit reduces the rents of satellite ownership without restricting entry, treating entrants and incumbents equally. These differing incentives can lead to welfare differences between stock and flow modes of orbit control. To show how stock and flow controls affect the decision to launch a satellite, consider two cases with price-based controls. In the first, a stock control is levied on satellite owners. In the second, a flow control is levied on satellite launchers. I assume the regulator can commit to future policies, so that $t+1$ values are known to firms with certainty.[10]

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[8] Keohane, Van Roy, and Zeckhauser (2007) consider the use of stock and flow controls to manage the quality of a resource, but their use of “stock control” is slightly different due to the setting considered. In their setting, “stock controls” refer to policies which restore the stock of a deteriorating resource. Here, the term refers to limiting the stock of a commodity which deteriorates the resource. Keohane, Van Roy, and Zeckhauser (2007)'s use of “flow controls” is closer to the use of the term here: they consider abating the flow of pollutants into the environment, and I consider controlling the flow of satellites into orbit.

[9] Provided capacity is not a binding constraint, production effort is costly, and the production function is not

The decision to launch under a stock control:

Similar leakage issues have been studied extensively in the environmental and public economics literatures, for example Fowlie (2009); Fischer and Fox (2012); Böhringer, Rosendahl, and Storrøsten (2017). Though these issues are relevant to effective policy implementation, analyzing them is beyond the scope of this paper. The legal hurdles to implementing stock controls may also be higher than those for flow controls, since they require a legal framework in which the right to exclude agents from an orbit can be held and enforced. Such a framework would have to be globally agreed-upon and potentially self-enforcing. I do not consider the prospects of such an agreement in this paper, although similar issues have been studied extensively in economics generally and environmental economics specifically, for example Telser (1980); Barrett (2005, 2013).

3.1.1 Using stock and flow space traffic control policies

In this section, I formally describe some properties of stock and flow controls and how they should be used to manage space traffic. The first property is price-quantity equivalence: under symmetric physical uncertainty, a stock or flow control can be implemented as a price or quantity and achieve equivalent expected social welfare. This allows me to consider price or quantity implementations interchangeably. I then show how stock and flow controls should be used to limit launches, and consider the implications of these details for optimal control values. I follow this by showing how the launch rate responds to the initiation of a stock or flow control, and how a regulator could use those controls to induce firms to deorbit already-orbiting satellites and stop launching new ones. These properties are used in the following section to establish that regulating orbit use through stock controls achieves higher expected until the price of a permit is

$$\tilde{p}_{t+1} : p = rF + E_t[\ell_{t+1}]F + \tilde{p}_{t+1} \tag{26}$$

For a given state vector $(S_t; D_t; \ell_t)$ and a chosen price p_{t+1}, the monotonicity of $E_t[\ell_{t+1}]$ ensures that equation 25 determines a unique value $\tilde{X}_t$. For the same state vector and $X_t = \tilde{X}_t$, the monotonicity of $E_t[\ell_{t+1}]$ ensures that $\tilde{p}_{t+1} = p_{t+1}$ solves 26.

Flow controls: I refer to price-based flow controls as launch taxes, and quantity-based stock controls as launch permit quotas. Let the launch rate in t under a launch tax be $\tilde{X}_t$, and the permit price in $t+1$ under a permit quota be $\tilde{p}_{t+1}$.

Under a launch tax, the number of satellites launched will be

$$\tilde{X}_t : p = rF + E_t[\ell_{t+1}]F + (1+r)p_t - (1 - E_t[\ell_{t+1}])p_{t+1} \tag{27}$$

Under a binding launch permit quota, firms will purchase permits and launch satellites until the price of a permit is

$$\tilde{p}_{t+1} : p = rF + E_t[\ell_{t+1}]F + (1+r)p_t - (1 - E_t[\ell_{t+1}])\tilde{p}_{t+1} \tag{28}$$

For a given state vector $(S_t; D_t; \ell_t)$ and a chosen price p_{t+1}, the monotonicity of $E_t[\ell_{t+1}]$ ensures that equation 27 determines a unique value $\tilde{X}_t$. For the same state vector and $X_t = \tilde{X}_t$, the monotonicity of $E_t[\ell_{t+1}]$ ensures that $\tilde{p}_{t+1} = p_{t+1}$ solves 28. □

With access to commitment, a regulator using a flow control sets either the future number of permits or their price (X_{t+1} or p_{t+1}) in order to influence the launch rate today (X_t). Raising p_{t+1} in t raises the marginal benefit of launching a satellite today, but lowers it tomorrow. The use of flow controls requires the regulator to trade off the future launch disincentive of raising p_{t+1} against the current launch incentive it creates. The regulator's true instrument with a flow control is not the price of the control itself, but the *change* in price between periods. Rather than a price mapping to a quantity, here it is a (real) change in price which maps to a quantity and vice versa. The regulator can set any initial flow control price so long as they commit to a path of control prices based on equation 24. A similar penalty-rebate structure appears in the mining flow control studied in Briggs (2011), where incentivizing mine owners to mine less in t requires a lower Pigouvian tax in period $t+1$.

Note that stock control prices must be positive to reduce launches in any given period, while flow control prices need not be positive to do the same. Along positive price paths the flow control is an entry restriction while along negative price paths it is an entry subsidy.

Current restrictions deter current entry, but future restrictions deter future entry and boost the rents accruing to incumbents, incentivizing current entry. Current subsidies encourage current entry, but future subsidies encourage future entry and reduce the rents accruing to incumbents, incentivizing rms to delay entry. In either case, the regulator is able to use the change in ow control prices to rearrange satellite launches over time.[11]

The need to commit to a ow control path makes terminal conditions economically relevant to their use. If the regulator plans to use the ow control for only a limited duration, after which the orbits will be under open access again, the ow control price path will decrease over time until it is zero in the period where open access is restored.[12] A ow control which attempts to ensure optimality with no planned phase-out will be forced to follow an exploding price path, positive or negative, as the regulator attempts to balance present and future incentives

Proof. See Appendix section 8. □

Corollary 1. The shift in marginal cost of owning a satellite due to an increase in the ow control price is greater than the prior shift in marginal bene t due to the entry restriction.

Proof.

$$r > 0 \Rightarrow \frac{\partial X_t}{\partial_t p} > \frac{\partial X_t}{\partial p_{t+1}} \tag{29}$$

$$\Rightarrow \quad 1 + r > 1 \quad E[\ell_{t+1}]; \tag{30}$$

which is true because ℓ_{t+1}

Inducing satellite owners to deorbit: Satellite owners often have the option to deorbit their satellite if it becomes too expensive to operate.[13] In this section only, I include the deorbit option for satellite owners to consider whether stock and ow control policies can induce deorbits. The rm's net payoff from deorbiting their satellite is V^d 7V_d

prices cannot make satellite owners deorbit their satellites or induce net deorbits.

Proof. See Appendix section 8. □

Intuitively, making it costlier for rms to launch new satellites cannot make already-orbiting satellites less valuable. This is why ow controls are unable to induce deorbits, at least with positive prices. Flow controls with negative prices may or may not be able to induce deorbits, depending on parameter values and the number of new entrants induced.

3.1.2 Risks and policy choice

In this section, I consider how the choice of stock or ow control mode will affect the equilibrium collision risk and the probability of Kessler Syndrome. I establish that stock controls generate weakly higher expected eet values than ow controls over arbitrary horizons. Due to Ke.Fmoothn345(l1

satellite owners.

The relative advantage of stocks vs. ows: The question of ultimate interest to a regulator is likely one of policy choice: “which type of instrument is better, and why?” The results so far - particularly Proposition 4 - suggest that stock controls should be preferred to ow controls along generic paths. Proposition 5 compiles the results so far to answer the policy choice question along optimal paths. Since stock controls can be initiated without losing control of the launch rate and induce deorbits when necessary, they can achieve rst-best outcomes in every state of the world. Flow controls cannot. Even if interior launch rates are optimal forever and no deorbits are ever required, ow controls will achieve less social welfare than stock controls when they are put into place.

Proposition 5. (The relative advantage of stocks vs. ows) The expected social welfare under an optimal stock control strictly exceeds the expected social welfare under an optimal ow control for an arbitrary horizon where a control must be initiated, used to stop all launches, or used to force net deorbits.

Proof. The eet welfare from both controls can be equal along interior equilibrium paths. However, when the ow control is initiated, Proposition 2 shows that it will cause the launch rate to exceed the uncontrolled open access launch rate whereas a stock control would not. Proposition 4 shows that the launch bunching from initiating a ow control will also cause the risk of Kessler Syndrome to increase. In those periods, stock controls will achieve strictly greater expected social welfare than ow controls.

Proposition 3 shows that ow controls may not be able to induce net deorbits (never with positive prices and only possibly with negative prices), while stock controls can always do so. Therefore, for arbitrary paths with positive prices where the regulator must either initiate control, shut down orbital access, or induce net deorbits, stock controls achieve strictly greater expected social welfare than ow controls. □

Proposition 5 is fairly straightforward, and may even understate the advantages of stock controls over ow controls. From a computational perspective, optimal ow controls are much harder to implement than optimal stock controls because they require attention to the entire control time path. Lemma 3 in the Appendix shows that price-based ow controls must have an exploding price path to balance the launch incentives and disincentives described in Lemma 1 and Figure 4. One solution to this may be to use a quantity ow control, such as a launch permit quota system. However, the regulator must still commit to a time path of quantity policies when the ow control is implemented, cannot prevent launch bunching before the policy goes into effect, and cannot induce deorbits. Stock controls face none of these issues. A one-period-forward forecast of the marginal external cost is suf cient, which would have been required anyway under a flow control. The regulator faces no commitment issues and can precisely control the number of satellites in orbit at any given time.

3.1.3 Optimal space traffic control policies

Before finishing my discussion of stock and flow controls, I illustrate optimal control policy functions by simulation. For clarity and computational tractability, I use deterministic simulations where $E_t[\ell_{t+1}] = L(S_{t+1}, D_{t+1})$. Figure 7 shows an example of optimal stock and flow control policies as a satellite tax and a launch tax. Figure 6 shows the underlying satellite stocks, debris stocks, and launch rates used to compute Figure 7.

The magnitudes of the tax policies should not be taken literally; the underlying model has not been calibrated to either economic returns or physical processes. The point of the figures is to show the qualitative properties of optimal stock and flow controls. While both types of tax vary with the marginal external cost of launching a satellite ($E_t[x(S_{t+1}, D_{t+1})]$), only the satellite tax varies positively with the marginal external cost. This is a convenient feature for applying stock controls: the behavior of an optimal stock control is more intuitive than that of an optimal flow control. The reasoning behind this behavior is described in in Lemma 1 and Figure 4.

[Figure 6 about here.]

[Figure 7 about here.]

3.2 Active debris removal and open access

I now turn to the effects of active debris removal technologies on orbit use. My main result, Proposition 6, shows that while active debris removal can mechanically reduce the debris stock no matter how it is financed, it can only reduce the equilibrium risk of satellite-destroying collisions to the extent that satellite owners pay for debris removal. I show this in two steps. First, I show that exogenously provided removal which is free to satellite owners will reduce the debris stock but increase the satellite stock. The increase in the satellite stock will exactly offset the decrease in risk from debris removal, leaving the equilibrium collision risk unchanged. Then, I consider a case where exogenous debris removal involves a mandatory fee paid by satellite owners. I show that as the fee goes to zero, the collision risk returns to the original open access level.

Lastly, I show how endogenously chosen debris removal purchased by cooperative satellite owners reduces the debris stock, collision risk, and risk of Kessler Syndrome, while also allowing more firms to launch satellites. These results depend on some auxiliary properties of cooperative debris removal and open access launching with debris removal, shown in the Appendix, section 6.7. Though the jointly-optimal launch and removal plan is analytically complicated, I simulate the eet planner's launch and removal plans and compare them to the launch and removal plans under open access and cooperative removal. The simulations show that cooperative decentralized removal plans are identical to the planner's removal plans, though the launch plans differ. The differences in the launch plans are particularly interesting: the eet planner launches more intensely than rms under open access do.

3.2.1 An economic model of active debris removal

Satellite owners purchase R_t

The value of a satellite owner who purchases debris removal before the loss is

$$\begin{aligned}
\tilde{Q}_i(S_t, D_t) &= \max_{0 \le R_{it} \le D_t/S_t} \{-c_t R_{it} + \tilde{E}_t[Q_i(S_t, D_t, \ell_t, X_t)]\} && (47)\\
\text{s.t. } \ell_t &\sim f(\ell_t | S_t, D_t - R_t)\\
Q_i(S_t, D_t, \ell_t, X_t) &= p + \beta[(1-\ell_t)\tilde{Q}_i(S_{t+1}, D_{t+1}) + \ell_t E_t[V_i(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})]]\\
S_{t+1} &= S_t(1-\ell_t) + X_t\\
D_{t+1} &= (D_t - R_t)(1-\delta) + G(S_t, D_t - R_t, \ell_t) + mX_t.
\end{aligned}$$

The value of a launcher is

$$V_i(S_t, D_t, \ell_t, X_t) = \max_{x_{it} \in \{0,1\}} \{(1-x_{it})\beta E_t[V_i(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] + x_{it}[\beta\tilde{Q}_i(S_{t+1}, D_{t+1}) - F]\} \quad (48)$$

$$\begin{aligned}
\text{s.t. } \tilde{Q}_i(S_t, D_t) &= \max_{0 \le R_{it} \le D_t/S_t} \{-c_t R_{it} + \tilde{E}_t[Q_i(S_t, D_t, \ell_t, X_t)]\}\\
\ell_t &\sim f(\ell_t | S_t, D_t - R_t)\\
Q_i(S_t, D_t, \ell_t, X_t) &= p + \beta[(1-\ell_t)\tilde{Q}_i(S_{t+1}, D_{t+1}) + \ell_t E_t[V_i(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})]]\\
S_{t+1} &= S_t(1-\ell_t) + X_t\\
D_{t+1} &= (D_t - R_t)(1-\delta) + G(S_t, D_t - R_t, \ell_t) + mX_t.
\end{aligned}$$

Under a generic launch plan, the decision to remove debris is dynamic. Removal today will impact the amount of debris tomorrow through the number of satellite destructions and the number of debris-debris collisions. Under open access, the value of a satellite tomorrow will always be driven down to the current value of the launch cost, so the future benefits of removal will never accrue to today's satellite owners. This makes the removal decision under open access static: the only benefit of debris removal internalized by satellite owners today is the way that it changes the probability that their satellite is destroyed. Even though the cost of removal is linear, nonlinearity in the coupling between the debris stock and the collision rate can yield an interior solution to the removal decision.

Open access launching: Under open access, firms will launch satellites until the value of launching is zero:

$$\begin{aligned}
\forall t, X_t : V_i(S_t, D_t, \ell_t, X_t) &= 0 && (49)\\
\implies \beta\tilde{Q}(S_{t+1}, D_{t+1}) &= F && (50)\\
\implies Q_i(S_t, D_t, \ell_t, X_t) &= p + (1-\ell_t)F. && (51)
\end{aligned}$$

Taking R_t as fixed, and assuming that launchers plan to choose R_{t+1} optimally when they are satellite owners, the ow condition determining the launch rate is

$$p = rF + \tilde{\tilde{E}}_{t+1}[`$$

If they could not coordinate as launchers, how can they do so as satellite owners? The answer

From equation 9, the equilibrium collision risk without debris removal[16] is

$$\tilde{E}_{t+1}[\ell_{t+1}] = r_s \quad r: \tag{59}$$

If no debris is removed,

of post-removal debris is a constant. As a result, even if there is an increase in the number of

The change in this probability due to an increase in R_t is

$$\frac{d\Pr_t(D_{t+1}(\varepsilon_t) - D^k > 0 | S_t; D_t - R_t)}{dR_t} = \frac{\partial}{\partial R_t} \int_0^1 1(D_{t+1}(\varepsilon_t) - D^k > 0) f(\varepsilon_t | S_t; D_t - R_t) d\varepsilon_t \tag{65}$$

$$= \int_0^1 1\left(\frac{\partial D_{t+1}(\varepsilon_t)}{\partial R_t} > 0\right) f(\varepsilon_t | S_t; D_t - R_t) d\varepsilon_t + \int_0^1 1(D_{t+1}(\varepsilon_t) - D^k > 0) \frac{\partial f(\varepsilon_t | S_t; D_t - R_t)}{\partial R_t} d\varepsilon_t \tag{66}$$

$$= \Pr_t\left(\frac{\partial D_{t+1}(\varepsilon_t)}{\partial R_t} > 0 \Big| S_t; D_t - R_t\right) - \left[\frac{\partial \Pr_t(D_{t+1}(\varepsilon_t) - D^k > 0 | S_t; \tilde{D} - R_t)}{\partial \tilde{D}}\right]_{\tilde{D} = D_t} . \tag{67}$$

The first term in equation 67

then

$$\frac{d\Pr_t(D_{t+1}(\ell_t) - D^k > 0 | S_t; D_t - R_t)}{dR_t} = -\frac{\partial \Pr_t(D_{t+1}(\ell_t) - D^k > 0 | S_t; \tilde{D} - R_t)}{\partial \tilde{D}}\bigg|_{\tilde{D}=D_t}. \quad (68)$$

The right hand side of equation 68 is the negative of the change in the probability of Kessler Syndrome from the shift in the distribution of collision rates which a marginal amount of debris would cause. It is not precisely the same as the effect of another unit of debris [redacted] the debris argument of $D_{t+1}(\ell_t)$ is held constant while the debris argument of $f(\ell_t|S_t; D$ [redacted] is increased slightly. By Assumption 1, increasing the amount of debris [redacted] the conditional density of the collision rate toward 1. The fact that $D_{t+1}(\ell$ [redacted] at least weakly increasing in ℓ_t, combined with Lemma 8, the change in p[redacted]ility must be at least weakly positive. So, debris removal must reduce the probability of Kessler Syndrome:

> -6786 [(9[()

$$\partial \Pr_t(D_{t+1}(\ell_t) - D^k > 0 | S_t; \tilde{D} - R_t) \; D \; 0 \; ;_{\tilde{D}=D_t}$$

$$\tilde{W}(S_t, D_t) = \max_{R_t \in [0, D_t]} \{ -c_t R_t + \tilde{E}_t[W(S_t, D_t - R_t, \ell_t)] \} \tag{71}$$

$$\text{s.t. } W(S_t, D_t - R_t, \ell_t) = \max_{X_t \geq 0} \{ pS_t - FX_t + \beta \tilde{W}(S_{t+1}, D_{t+1}) \}$$

$$\ell_t \sim f(\ell_t | S_t, D_t - R_t)$$

$$S_{t+1} = S_t(1 - \ell_t) + X_t$$

$$D_{t+1} = (D_t - R_t)(1 - \delta) + G(S_t, D_t - R_t, \ell_t) + mX_t.$$

The planner faces the same timing of information as firms do: at the beginning of a period, before ℓ_t has been revealed, they choose how much debris they will remove. Based on their removal decision, the draw of ℓ_t is revealed. Then they decide how much they will launch. The program in system of equations 71 shows this decision-making process at the beginning of a period. Their jointly-optimal removal and launch plans must equate the social marginal costs and benefits of removing debris before ℓ_t is known and of launching satellites once ℓ_t is known. Formally,

$$R_t : c_t = \left\{ -\tilde{E}_t \frac{\partial W(S_t, D_t - R_t, \ell_t)}{\partial D_t} + \frac{\partial \tilde{E}_t[W(S_t, D_t - R_t, \ell_t | S_t, D - R_t)]}{\partial D} \right\}_{D = D_t} \tag{72}$$

$$X_t : \frac{F}{\beta} = \frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial S_{t+1}} + m \frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial D_{t+1}} \tag{73}$$

An optimal removal plan exists if the sum of the objects inside the curly brackets on the right side of equation 72 is positive. I assume that the marginal post-removal value of debris is negative ($\partial W(S_t, D_t - R_t, \ell_t$

[Figure 10 about here.]

Comparing Figure 10 with Figure 3 shows that the open access launch plan with debris removal is similar to the plan without debris removal given cooperative debris removal. With removal, however, there is a jump in the launch rate just as it becomes optimal for cooperative satellite owners to begin removing debris. This jump is shown in the time paths in Figure 9. This is because debris removal by incumbent satellite owners allows new firms to enter the orbit. Since the planner keeps the debris stock at a constant level as soon as the fleet value justifies it, they ignore debris while launching. More formally, controlling both satellite launches and debris removal allow the optimal policies to be piecewise-concave in satellites and debris.

The cooperative debris removal plan and the planner's removal plan are both corner solutions once debris removal starts.[17] The planner, however, begins debris removal with fewer satellites than the cooperative firms. Intuitively, the planner starts removing debris once the fleet is valuable enough to justify removal, while cooperative satellite owners start removing debris once there are enough owners sharing the removal costs to justify removal.

The discontinuity in the open access launch plan, its dependence on the debris stock, and the later start in the cooperative debris removal plan all reduce the open access-cooperative fleet value relative to the fleet planner's. The value loss from open access launching and cooperative debris removal follows the launch plan deviation and is intensified along the removal plan deviation. The gap is maximized just before open access launchers, anticipating removal, begin to launch again. At that point, the planner would have stopped launching and have begun removing debris while cooperative satellite owners would still be waiting for more contributors.

4 Conclusion

In this paper I showed how principles of economics should guide our stewardship of orbital resources. I established the equivalence of price and quantity instruments for orbital management and showed why space traffic controls should target satellite ownership rather than satellite launches. I considered the impacts of using active debris removal technology, and showed why, to reduce equilibrium collision risk, satellite owners must pay for debris removal.

[17] See Appendix section 6.5 for more details on nonconvexities and corner solutions in debris removal.

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5 Appendix A: Institutional details of orbit use

5.1 International laws regarding space traf c control

Orbits are inherently global resources, and space law is fragmented across nations and documents. Space law spans domestic policies, international treaties, bilateral agreements, and guidelines. Not all agreements are signed by all spacefaring nations, and many are non-binding. Most of the agreements are vague and suffer from enforcement problems. Four of the most relevant international agreements relating to orbit management are the 1967 Outer Space Treaty, the 1972 Liability Convention, the 1975 Registration Convention, and the 2007 COPUOS Guidelines.[18]

1967 Outer Space Treaty The Outer Space Treaty[19] established the legal framework for peaceful uses of outer space. Article 2 of the Treaty designates outer and orbital space as common pool resources, to be used "for the bene t of all" humankind. The only explicit restrictions are on military uses and claims of national sovereignty; the state of resource use is left ambiguous. The Treaty does not mention debris, only stating that nations should avoid causing (unde ned) "harmful contamination" of outer space.

1972 Liability Convention The Liability Convention[20] established the framework for tort law of space activities. However, the Convention focused more on damage to terrestrial objects from re-entry than on damages to orbital objects which occur in space. "Damage" in this Convention is de ned only in relation to realized outcomes for people and property, rather than potential outcomes caused by the environment. Additionally, the Convention places liability for such damages on the launching state rather than the launching entity. This has motivated nations like the US to require satellite owners insure their satellites, with the federal government indemnifying losses beyond a certain amount. The EU has different insurance requirements, with a similar motivation. There is no liability attached to producing debris in orbit, only to attributable damages. Liability extends to damage to people or property caused by re-entry. Such attribution is dif cult in space, where damages may be caused by dif cult-to-detect fragments of unknown origin.

1975 Registration Convention The Registration Convention[21] requires nations to register space objects launched from or by that nation with the UN Secretary-General. The responsibility for ensuring compliance lies with the launching state, with the UN being responsible for integrating all the registrations and publishing a publicly available international registry of

[18] A more detailed analysis of these laws can be found in Akers (2012).

[19] The "Treaty on Principles Governing the Activities of States in the Exploration and Use of Outer Space, Including the Moon and Other Celestial Bodies."

[20] The "Convention on International Liability for Damage Caused by Space Objects."

[21] The "Convention on Registration of Objects Launched into Outer Space."

objects in orbit. The Convention only requires basic orbital information to be provided: orbital parameters to ascertain the object's initial path, and the general function. It does not require

Article 4 of the Outer Space Treaty declares that state parties “undertake not to place in orbit around the Earth any objects carrying nuclear weapons or any other kinds of weapons of mass destruction, install such weapons on celestial bodies, or station such weapons in outer space in any manner.” It also forbids establishing military installations, conducting weapons tests, or any other non-peaceful activities on the Moon and other celestial bodies. Despite these provisions, the Outer Space Treaty does not explicitly prohibit using near-Earth space for reconnaissance, terrestrial warfare coordination, or even outright con ict so long as “weapons of mass destruction” are not used in orbit. The ongoing militarization of space has therefore involved these uses, with the US government being the largest such user of orbital space. The US government has not yet supported international treaty efforts to limit the militarization of space. Shimabukuro (2014) offers an explanation for the lack of more international regulation is uniformly decreasing in the collision risk draw, and a third where it is uniformly increasing in the collision risk draw. There are two competing effects of collisions driving this behavior: collisions generate debris, but collisions also remove other satellites from orbit.

Proof. Cooperation with a non-zero debris removal plan is robust to all deviations ϵ for which

$$\tilde{E}_t[\ell_t|S_t;D_t+\epsilon-R_t]-\tilde{E}_t[\ell_t|S_t;D_t-R_t]>\epsilon\frac{c_t}{F}.$$

Cooperation with a non-zero debris removal plan strictly dominates small deviations if

$$\lim_{\epsilon\to 0}\frac{\tilde{E}_t[\ell_t|S_t;D_t+\epsilon-R_t]-\tilde{E}_t[\ell_t|S_t;D_t-R_t]}{\epsilon}>\frac{c_t}{F}\quad\forall R_t>0$$

$$\implies\left.\frac{\partial\tilde{E}[\ell_t|S_t;\bar{D}-R_t]}{\partial D}\right|_{\bar{D}=D_t}>\frac{c_t}{F}\quad\forall R_t>0.$$

□

The following proposition establishes that the debris removal solution described in equation 54 is in fact the cooperative private debris removal solution.

Proposition 10. (A cooperative private removal plan) The debris removal solution described by equation 54 is maximizes the value of the currently-orbiting satellite fleet, given open access in that period.

Proof. Given open access launch rates, the value of a satellite already in orbit is

$$Q_i(S,D)=p-cR_i+(1-\tilde{E}[\ell])F.$$

Given open access launch rates, the value of all satellites already in orbit is

$$\begin{aligned}W(S,D)&=\int_0^S Q_i\,di\\&=pS-cR+(1-\tilde{E}[\ell])FS.\end{aligned}$$

Equation 54 is the first-order condition for the firm's problem,

$$Q_i(S,D)=\max_{0\le R_i\le D/S}\{p-cR_i+(1-\tilde{E}[\ell])F\}.$$

A constrained planner R+(1 − Ẽ[ℓ])m-367(]6 7.9701 Tf 3.m6 7.9701 T02rm'01 T250(of)-25001 T)-25368T02rm'01 individual removal solution given by equation 54 therefore characterizes a cooperative debris removal solution, where each rm behaves as an open-access-constrained social planner would command. □

6.4 Competitive debris removal pricing

The pro ts of the cleanup industry, which supplies active debris removal, are

$$I_t(R_t) = \underbrace{c_t R_t}_{\text{Cleaning revenues}} \quad \underbrace{g R_t^2}_{\text{Cleaning costs}} : \tag{81}$$

If the cleanup industry is competitive and a positive amount of debris is removed, debris will be removed from orbit until industry pro ts are zero, between satellites and debris possible.

When the satellite and debris couplings in the collision rate depend on each other, that is, $L_{SD} \neq 0$, changes in the satellite stock can change the returns to scale for debris removal. The dynamic benefits of debris abatement also include the effect of fragment growth from collisions between debris. This effect implies that the net marginal rate of debris decay $(d - G_D(S, D - R))$ can be negative.

The marginal benefit of removal is the private value of reducing the probability of a satellite-destroying collision. Debris removal has diminishing marginal benefits if and only if the collision rate is strictly convex in debris. The upper bound on $L(S, D)$ implies that debris removal will have increasing marginal benefits when the risk of a collision gets high enough. Figure 12 shows two examples of this, one with a negative exponential collision rate (globally concave) and another with a sigmoid collision rate (first convex and later concave).

[Figure 12 about here.]

For any positive initial level of debris and satellites (S, D), removal must be nonnegative and no more than all of the debris can be removed. When all satellite owners are identical, the maximum that any one can remove is D/S. This closes the feasible set. Any intermediate amount can also be removed, making the feasible set convex.

The nonconvexity of marginal removal benefits complicates analysis of the optimal amount of removal. There are two cases: the collision rate is globally concave, or the collision rate is convex over some nonnegative interval.

1. If the collision rate is globally concave, there can be no interior solution to the satellite owner's removal problem. Gllisi6n1.

second-order condition (inequality 55), and include them in a set with zero removal and full removal. This is the set of candidate solutions. Calculate the profits of each candidate solution, and select the one with the highest profits. This procedure is computationally tractable over a closed and convex support as long as the collision rate function is reasonably well-behaved. Figure 13 illustrates how nonconvexity of the collision rate affects profits and the optimal level of removal.

[Figure 13 about here.]

6.6 Cost and congestion shifts in cooperative removal demands from new satellites

For brevity, I write $E_t[\ell_{t+1}]$ as $L(S_t; D_t \;\; S_t R_{it})$ in this subsection and use S and D subscripts to indicate the respective partial derivatives. Since these results are intratemporal in nature, I also drop time subscripts.

To see the cost and congestion scollisi39(I)-237(writin-190(cost)-190(an(v)10(e)-230(partia190(p(ati)25(

would increase the effect of a marginal unit of debris, the congestion shift will be positive. Reducing the amount of debris would greatly reduce the threat posed by the marginal satellite. If a marginal Unkind would decrease the effect of a marginal unit of debris, the congestion shift will be negative. This could be the case if the Unkind was well-shielded from debris but a threat to other satellites. Reducing the amount of debris would not change the risk of the marginal Unkind by much then.

The cost shift is the effect of increasing the number of customers in the market for debris removal on the marginal collision threat from a unit of debris. There are two pieces to this. First, the debris removed by each Kind reduces the collision risk for all owners. As long as the collision rate is increasing in debris, reducing debris is always a good thing for everyone. This will tend to make the cost shift positive. Second, the debris removed by each Kind changes the marginal bene t of the next Kind's removal. Since the collision rate must be locally convex in debris at an interior solution, this effect will tend to make the cost shift negative. If the collision rate is suf ciently locally convex, this effect can make the cost shift negative in total. Generic satellites are both Kinds and Unkinds.

6.7 Comparative statics of cooperative debris removal and open access launching

I show three results about the demand for debris removal in this section.

First, there is a unique cooperatively-optimal post-removal level of debris for any given level of the satellite stock. This is a consequence of the linear cost (to satellite owners) of debris removal and the monotonicity of the expected collision risk in debris. Due to the linearity, cooperative satellite owners will pursue a most-rapid approach path to the optimal post-removal level of debris in every period. Were the cost nonlinear, the most-rapid approach path would no longer be optimal but the optimal level of debris would remain unique due to monotonicity.

Second, if satellites and debris are “strong enough” complements in producing collision risk, increasing the number of satellite owners in orbit will reduce the optimal post-removal level of debris. This spillover effect in debris removal suggests that a “dynamic virtuous cycle” of active debris removal may be possible: removal in one period can spur entry in the next, which in turn spurs more removal in the following period. Although the functional forms I use rule this effect out, those forms are simpli ed from a statistical mechanics approximation of

Third, the open access launch rate may be *increasing* in the launch cost. Though this result seems counterintuitive, it is a natural consequence of three features of open access orbit use:

1. open access drives the value of a satellite down to the launch cost;
2. the amount of removal is increasing in the launch cost;
3. new entry can reduce the individual expenditure required from cooperative firms to achieve the optimal post-removal level of debris.

The cooperative cost-savings from new entry exceeding the effect of new entry on collision risk is necessary and sufficient for the open access launch rate to be increasing in the launch cost.

Together, these results suggest that the use of debris removal can result in interesting and counterintuitive dynamics in orbit use. Though these results are relevant to understanding the effects of debris removal technologies on orbit use, I omit their proofs from this section. Interested readers may find the proofs in the Appendix, section 8.

Cooperative private debris removal:

Lemma 5. (Law of cooperative private debris removal demand) The cooperative private debris removal demand is

1. weakly decreasing in the price of removing a unit of debris, and
2. weakly increasing in the cost of launching a satellite.

Proof. I consider corner solutions first, then interior solutions. I characterize how interior solutions change in response to a change in the removal price, then show that increases in the price can only induce the firm to reduce their removal demands even at corners. I refer to the non-optimized value of a satellite as $Q_i(R_i)$.

The full removal corner: The first part of the proposition is trivially true at the full removal corner, since the amount of debris removal purchased cannot increase at this corner. So it must either stay the same, or decrease, in response to an increase in the price of removal. For the second part, suppose a firm initially finds full removal optimal. Reducing the amount of debris by a positive amount ϵ in response to a change in launch cost removed is optimal if and only if, at the new launch cost,

$$Q_i(D/S - e) - Q_i(D/S) > 0 \quad \forall e \in \left(0, \frac{D}{S}\right]$$

$$\implies p + F - c\frac{D}{S} + ce - \tilde{E}[\ell|S, D - S(D/S - e)]F - p - F + c\frac{D}{S} + \tilde{E}[\ell|S, 0]F > 0$$

$$\implies ce - (\tilde{E}[\ell|S, e] - \tilde{E}[\ell|S, 0])F > 0$$

$$\implies \frac{\tilde{E}[\ell|S, e] - \tilde{E}[\ell|S, 0]}{e} < \frac{c}{F} \quad \forall e \in \left(0, \frac{D}{S}\right].$$

If full removal was optimal to begin with, then an increase in the launch cost cannot make it optimal to switch strategies. The above inequality also shows how an increase in the cost of removal can induce a firm to reduce the amount of removal purchased.

The zero removal corner: Consider the profits from increasing the amount of removal from zero to e

another, they must jump to a solution with less removal.

Similarly, from applying the Implicit Function Theorem to H,

$$\frac{\partial R_i}{\partial F} = -\frac{\partial H/\partial D}{\partial H/\partial R_i}$$
$$= \frac{\frac{\partial \tilde{E}[\ell]}{\partial D} S}{\frac{\partial^2 \tilde{E}[\ell]}{\partial D^2} S^2 F} > 0.$$

Strict positivity follows from the second order condition (inequality 55). If there are multiple solutions and the launch cost increase causes firms to jump from interior one solution to another, they must jump to a solution with more removal. □

The intuition for this result is simple. Satellite owners pay for debris removal. When the price of removal rises, the demand for removal falls. Under open access the continuation value of a satellite is the cost of launching. So, the demand for debris removal increases when satellites become more valuable. Figure 14 illustrates Lemma 5.

[Figure 14 about here.]

Applying the Implicit Function Theorem to H ,

¶ R_{it}

R_{it} and S_t are both nonnegative by definition. It follows that

$$\frac{\partial R_t}{\partial S_t} > 0 \iff \frac{R_{it}}{S_t} > \frac{\partial R_{it}}{\partial S_t}:$$

This is always true when individual removal demands increase in response to additional satellite owners ($\frac{\partial R_{it}}{\partial S_t} > 0$). The following steps establish the complementarity condition for interior solutions.

From equation 54,

$$R_{it} : H = c_t \quad \frac{\partial \tilde{E}_t[\ell_t | S_t; D_t \quad R_t]}{\partial D_t} S_t F = 0:$$

Applying the Implicit Function Theorem to H,

$$\frac{\partial R_{it}}{\partial S_t} = \frac{\partial H = \partial S_t}{\partial H = \partial R_{it}}$$

$$= \frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S_t^2} + \frac{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t} \quad \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} R_{it}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S_t} \gtrless 0:$$

So, increases in the amount of debris must increase the privately optimal amount of removal at all interior solutions, while increases in the number of satellites will have ambiguous effects. The privately optimal demand for removal will be increasing in the number of satellites if and only if

$$\frac{\partial R_{it}}{\partial S_t} > 0 \iff \frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S_t^2} + \frac{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t} \quad \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} R_i}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S} > 0$$

$$\iff \frac{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S} > \frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S^2} + \frac{R_i}{S}$$

$$\iff \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t} < \frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{S} + \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} R_i:$$

The right hand side of the nal line is strictly negative Td [(0 Td [(`)]TJ/F91 5.9776 oe)15(gt)]TJ/F96 17.9701 but the contribution required of each rm decreases when more rms enter. At an interior solution, their response to more satellites depends on two effects: a congestion effect and a cooperation effect. Their net effect depends on the collision rate's convexity in debris and satellites, particularly whether satellites and debris are “strong enough” complements in producing collision risk. If

Indeed, this is precisely what occurs in the cases simulated here.

In addition to this perhaps-counterintuitive effect, it is plausible that an increase in the cost of launching a satellite could increase the launch rate. This is not as pathological a case as it may seem at first. Since open access drives the value of a satellite down to the launch cost, and the cooperatively-optimal amount of debris removal which satellite owners will pay for is increasing in the launch cost, and increase in the launch cost under open access could increase the value of owning a satellite by more than it increases the cost of launching it, at least locally near an existing equilibrium. This is not a violation of the law of demand for satellite ownership; rather, it is a violation of the "all else equal" clause. Assumption 4 describes a necessary and sufficient condition to rule this case out.

Assumption 4. (New launches reduce the expected profits of satellite ownership) The change in individual removal expenses from a marginal satellite launch is smaller in magnitude than the sum of the change in expected future collision costs from a marginal satellite launch and the change in individual removal expenses from a marginal piece of launch debris. Formally,

$$\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} F + \mu \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}} F + \frac{\partial R_{it+1}}{\partial D_{t+1}} c_{t+1} > \frac{\partial R_{it+1}}{\partial S_{t+1}} c_{t+1} .$$

If this assumption is violated, then launches increase the profitability of owning a satellite through the debris removal expenditure channel described above. It is also possible that increases in the cost to satellite owners of removing a unit of debris could increase the launch rate Assumption 5 describes an additional condition necessary for increases in the price of debris removal to reduce the launch rate. An increase in the price of debris removal will reduce the cooperatively-optimal amount of debris removal satellite owners purchase, potentially reducing the total debris removal expenditure and increasing the profits of owning a satellite. As in the case of launch rates being increasing in launch costs, this is not a violation of the law of demand for satellite ownership; it is a violation of the "all else equal" clause.

Assumption 5. (Removal expenditure is increasing in the removal cost) The cooperative private debris removal expenditure is increasing in the price of removing a unit of debris. Formally,

$$\frac{\partial}{\partial c_{t+1}}(R_{it+1} c_{t+1}) = R_{it+1} + \frac{\partial R_{it+1}}{\partial c_{t+1}} c_{t+1} > 0.$$

Assumption 5 states that the amount of debris removed (R_{it+1}) is larger than the reduction in removal due to a price increase ($\frac{\partial R_{it+1}}{\partial D_{t+1}} c_{t+1}$, which is weakly negative from Proposition 5). This is likely to hold whenever the change in individual removal demands from a change in removal cost is small, for example,if removal demand is in the interior before and after the change. It is unlikely to hold if the opposite is true, for example,if the change in removal cost causes individual removal demands to jump from the full removal corner to the zero removal corner at a time when there are few satellites and many debris fragments. Though future cooperative private debris removal demands are an anticipated cost to current satellite launchers, those same launchers may find their willingness to launch increasing in the cost of removal if it reduces the burden of cooperating and purchasing removal.

Proposition 13. (Private demand for satellite ownership) The open access launch rate is

1. strictly decreasing in the cost of launching a satellite if and only if new launches reduce the expected profits of satellite ownership; and

2.

7 Appendix C: Model extensions

7.1 Spectrum use management

So far I have assumed that there is no spectrum congestion from satellites. In practice, radio frequency interference is one of the major concerns of space traffic control. However, policy to manage spectrum use is not a focus of this paper because it is generally handled well by existing institutions Johnson (2004). The effect of optimally managed spectrum congestion on the expected collision risk in a deterministic setting is shown in the appendix of Rao and Rondina (2018). In this section, I adapt the result to this paper's setting and show that permits or fees for spectrum use can approximate stock controls.

Spectrum congestion degrades the quality of the signals to and from satellites. This makes the per-period output from a satellite decreasing in the number of orbital spectrum users. For simplicity, suppose that all satellites in orbit use enough spectrum to have some congestion impact. The per-period return function is then $\pi = p(S); p'(S) < 0$, and the one-period rate of return on a satellite is $p(S)/F = r_s(S)$. Assuming spectrum is optimally managed, firms will account for their marginal impact on spectrum congestion when they launch their satellite. The open access equilibrium condition, equation 9, becomes

$$E_t[\ell_{t+1}] = r_s(S_{t+1}) - r + r_s'(S_{t+1}). \tag{92}$$

Satellite owners would internalize the final term, $r_s'(S_{t+1})$, through a permit or fee system. Though spectrum permits may be purchased before the satellite is launched, their continued use is contingent on the firm abiding by non-interference protocols and any other stipulations by the appropriate regulatory body. Similarly, an optimal fee for spectrum use would adjust to reflect the marginal spectrum congestion from another broadcasting satellite. In general, regulated spectrum use will adjust the equilibrium collision rate to be

$$E_t[\ell_{t+1}] = r_s(S_{t+1}) - r - q_{t+1}; \tag{93}$$

where q_{t+1} is the spectrum use fee or permit price. Note that equation 93 is similar to equation 21,

$$p = rF + E_t[\ell_{t+1}]F + p^s_{t+1}$$
$$\implies E_t[\ell_{t+1}] = r_s - r - \frac{p^s_{t+1}}{F}. \tag{94}$$

This suggests another avenue for controlling the equilibrium collision rate. By setting the price of spectrum use, q_{t+1}, equal to the sum of marginal spectrum and collision risk congestion costs $-r_s'(S_{t+1}) + E_t[x(S_{t+1}; D_{t+1})]$, a spectrum regulator can implement an optimal stock control. More generally, this would be an optimal space traffic control in the sense of Johnson (2004), as it would account for both radio frequency and physical interference.

7.2 Mandatory satellite insurance

Can insurance markets correct the orbital congestion externality in the absence of active debris removal? Suppose that satellites were required to be fully insured against loss once they reached orbit and the satellite insurance sector was perfectly competitive. The insurance payment will act as a stock control, so the only question remaining is how the insurance industry will price the product. Denote the price of insurance in period t by p_t, and the profits of the insurance sector by I.

$$Q(S_t, D_t, \ell_t, p_t) = \pi - \underbrace{p_t}_{\text{Insurance premium}} + (1 - \ell_t)F + \underbrace{\ell_t F}_{\text{Insurance payout}} = \pi - p_t + F \tag{95}$$

$$I(S_t, \ell_t) = \underbrace{p_t S_t}_{\substack{\text{Inflow of} \\ \text{premium payments}}} - \underbrace{\ell_t S_t F}_{\substack{\text{Outflow of} \\ \text{reimbursements}}}. \tag{96}$$

Competitive insurance pricing With competitive insurance pricing, satellite insurance will be actuarially fair. Plugging this price into the open access equilibrium condition, we can solve for the loss rate under mandatory insurance:

$$p_t : I(S_t, \ell_t) = 0 \implies p_t = \ell_t F \tag{97}$$

$$\pi = rF + E_t[\ell_{t+1}]F + p_{t+1} \tag{98}$$

$$\implies E_t[\ell_{t+1}] = r_s - r. \tag{99}$$

Proposition 14. (Competitive insurance won't change collision risk) *The equilibrium collision risk given mandatory satellite insurance with actuarially fair pricing is the same as the equilibrium collision risk given uninsured open access.*

Proof. From equation 99, the equilibrium expected loss rate with actuarially fair insurance is

$$E_t[\ell_{t+1}] = r_s - r.$$

From equation 49, the equilibrium expected loss rate with no insurance is

$$E_t[\ell_{t+1}] = r_s - r.$$

□

Regulated insurance pricing As in the case of spectrum management policies, mandatory satellite insurance premiums approximate a stock control. This suggests another avenue by which a regulator could induce optimal orbit use without assigning property rights over orbits or levying an explicit satellite tax.

Suppose the regulator was able to give insurers a per-satellite penalty or subsidy of τ_t to ensure insurance would be priced at the marginal external cost $p_{t+1} - (1 - E_t[x(S_{t+1}; D_{t+1})])$ while still allowing free entry into the insurance sector. When τ_t is positive the regulator would be issuing an underwriting subsidy, and when τ_t is negative the regulator would be issuing an underwriting penalty. The insurance sector's profit is then

$$I(S_t; \ell_t) = (E_{t-1}[x(S_t; D_t)] - \ell_t F + \tau_t) S_t \tag{100}$$

$$\tau_t : I(S_t; \ell_t) = 0 \implies \tau_t = \ell_t F - E_{t-1}[x(S_t; D_t)]: \tag{101}$$

Equation 101 shows that the socially optimal mandatory insurance pricing can be achieved by an incentive which imposes the difference between the actuarial cost of satellite insurance and the marginal external cost on the insurer. The insurer then passes the marginal external cost on to the satellite owner. Depending on the magnitude of the risk and the marginal external cost, this may be a net subsidy or tax on the insurer.

8 Appendix D: Proofs and technical details

8.1 Proofs not shown in the main text

Lemma 1 (Launch response to stock and flow controls): The open access launch rate is

- decreasing in the future price of a stock control;
- decreasing in the current price and increasing in the future price of a flow control.

Proof. Stock controls: From equation 21, we can write

$$I = p - rF - E_t[\ell_{t+1}]F - p_{t+1} = 0: \tag{102}$$

Applying the Implicit Function Theorem, we get that

$$\frac{\partial X_t}{\partial p_{t+1}} = -\frac{\partial I/\partial p_{t+1}}{\partial I/\partial X_t} \tag{103}$$

$$= \frac{1}{\frac{\partial E[\ell_{t+1}]}{\partial X_t}} \tag{104}$$

$$= \frac{1}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial X_t} + \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial X_t}} \tag{105}$$

$$= \frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} + m\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}} < 0. \tag{106}$$

Flow controls: From equation 24, we can write

$$G = p - rF - E_t[\ell_{t+1}]F - (1+r)p_t + E_t[(1-\ell_{t+1})p_{t+1}] = 0. \tag{107}$$

Applying the Implicit Function Theorem, we get that

$$\frac{\partial X_t}{\partial p_t} = -\frac{\partial G/\partial p_t}{\partial G/\partial X_t} \tag{108}$$

$$= \frac{(1+r)}{\frac{\partial E_t[\ell_{t+1}]}{\partial X_t}}$$

Proof. In both cases, I suppose that there is open access before the control is initiated. Initiating a stock control: Suppose a stock control is scheduled to take effect at t, that is, satellite owners in t begin paying p_t. In $t-1$, firms would launch with this fact in mind:

$$X_{t-1} : p = rF + E_{t-1}[\ell_t]F + p_t \, : \tag{116}$$

Let the open access launch rate in $t-1$ with no stock control in t be $\hat{X}_{t-1} : p = rF + E_{t-1}[\ell_t]F$. Lemma 1 implies that for all $p_t > 0$, $\hat{X}_{t-1} > X_{t-1}$.

Initiating a flow control: Suppose a flow control is scheduled to be implemented at t, that is, satellite launchers in t begin paying p_t to launch. In $t-1$, firms would launch with this fact in mind:

$$X_{t-1} : p = rF + E_{t-1}[\ell_t]F - (1 - E_{t-1}[\ell_t])p_t \, : \tag{117}$$

Let the open access launch rate in $t-1$ with no flow control implemented in t be $\hat{X}_{t-1} : p = rF + E_{t-1}[\ell_t]F$. Lemma 1 implies that for all $p_t > 0$, $\hat{X}_{t-1} < X_{t-1}$.

□

Proposition 3 (Controlling the rate of deorbit): Stock controls with positive prices can make satellite owners deorbit their satellites and induce net deorbits. Flow controls with positive prices cannot make satellite owners deorbit their satellites or induce net deorbits.

Proof. A satellite owner facing a stock control in period t will deorbit if

$$p_t > p + (1 - E_t[\ell_{t+1}])F - V^d \, : \tag{118}$$

The regulator can induce firms to deorbit in t by raising p_t high enough in $t-1$. A potential launcher in $t-1$ will not launch if

$$p_t > p + (1 - E_{t-1}[\ell_t])F \, : \tag{119}$$

By raising p_t high enough, the regulator can both discourage further launches and induce existing satellite owners to deorbit their satellites.

A satellite owner facing a flow control in period t will deorbit if

$$p_t : p + (1 - \ell_t) b E_t[Q_{t+1}] < V^d; \text{ where } X_t : b E_t[Q_{t+1}] = F + p_t \tag{120}$$

$$\implies (1 - \ell_t)(F + p_t) < V^d - p; \tag{121}$$

which cannot be satisfied by positive p_t, given $V^d < p$. A potential launcher in t will not launch if

$$p_{t+1}: p + (1 - E_t[\ell_{t+1}])p_{t+1} < rF + E_t[\ell_{t+1}]F + (1+r)p_t. \quad (122)$$

If $p < rF + E_t[\ell_{t+1}]F + (1+r)p_t$, equation 122 will not be satisfied for any positive p_{t+1}. Although equation 122 can be satisfied if p_{t+1} is sufficiently negative, this would require the regulator to commit to a path of ever-decreasing negative prices as long as they wished to prevent launches (as described earlier and in Lemma 3). Regardless, the regulator cannot induce net deorbits (no new arrivals and some deorbits) in $t+1$ with a positive p_{t+1}. □

Proposition 7 (ADR can reduce collision risk):

Proof. I show the result first for the introduction of debris removal services, then for the ongoing use of debris removal services.

The introduction of ADR. Suppose an active debris removal service will become available at date t. To clarify whether removal is an option or not, I explicitly include the conditioning variables in the loss rate, that is, $E_t[\ell_{t+1}]$ is written as $E_t[\ell_{t+1}|S_t, p]$ is written as for ongoing debris removal to continue to reduce the collision risk:

$$E_{t-1}[\ell_t] - E_t[\ell_{t+1}] > 0 \iff \left(r_s - r - \frac{c_t}{F}R_{it}\right) - \left(r_s - r - \frac{c_{t+1}}{F}R_{it+1}\right) > 0$$
$$\iff c_{t+1}R_{it+1} > c_t R_{it}.$$

□

8.2 Technical assumptions and lemmas

Assumption 6. Let S be a vector of state variables, and I_k be a vector of same size as S with 1 in the k^{th} position and 0 in all other positions. $f(\ell|S)$ is a conditional density which satisfies the following properties:

1. The derivative of $f(\ell|S)$ with respect to the k^{th} argument of S,

$$\frac{\partial f(\ell|S)}{\partial S_k} = \lim_{h\to 0}\frac{f(\ell|S+I_k h) - f(\ell|S)}{h}$$ þo69e2291 1.66m85 0 7Td 281. [()99h4((xo4s:)]TJ 0 .66m85 0

changes to the collision probability, but that those changes will be bounded across the possible outcomes. This is economically reasonable for satellites - a violation of this implies that firms are deliberately placing their satellites in risky orbits. This may be less reasonable for debris, since the orbits of debris objects resulting from collisions are uncontrolled and difficult to predict. These conditions facilitate the proofs of the lemmas below, but are not crucial to the main results of the paper.

Note that the proofs of the lemmas below often assume uniformly bounded functions. While no such property is proven for the value functions studied, realistic economically sensible parameter choices should guarantee the existence of uniform bounds on the value functions.

Lemma 6. (Measurable functions under changes in distribution) Let ℓ be a random variable with a conditional density $f(\ell|S)$ defined on the compact interval $[a, b]$ and with range $[r(a), r(b)]$. Let $f(\cdot) : [r(a), r(b)] \to [f(a), f(b)]$ be a measurable function of ℓ. Then

$$\int_a^b f(\ell)\frac{\partial f(\ell|S)}{\partial S}d\ell = \frac{\partial E[f(\ell)|S]}{\partial S}$$

Proof.

$$\begin{aligned}
\int_a^b f(\ell)\frac{\partial f(\ell|S)}{\partial S}d\ell &= \int_a^b f(\ell)\lim_{h\to 0}\frac{f(\ell|S+h) - f(\ell|S)}{h}d\ell \\
&= \lim_{h\to 0}\frac{1}{h}\left[\int_a^b f(\ell)f(\ell|S+h)d\ell - \int_a^b f(\ell)f(\ell|S)d\ell\right] \\
&= \lim_{h\to 0}\frac{1}{h}(E[f(\ell)|S+h] - E[f(\ell)|S]) \\
&= \frac{\partial E[f(\ell)|S]}{\partial S}.
\end{aligned}$$

□

Lemma 7. $\frac{\partial E[f(x)|S]}{\partial S} = 0$ $\forall S$ and $\forall f(x)$ which do not depend on ℓ, the argument of $f(\ell|S)$.

Proof. From Assumption 6 and Lemma 6,

$$\begin{aligned}
\frac{\partial E[f(x)|S]}{\partial S} &= \int_0^1 f(x)\left(\lim_{h\to 0}\frac{f(\ell|S+h) - f(\ell|S)}{h}\right)d\ell \\
&= f(x)\lim_{h\to 0}\frac{1}{h}\left[\int_0^1 f(\ell|S+h)d\ell - \int_0^1 f(\ell|S)d\ell\right] \\
&= f(x)\lim_{h\to 0}\frac{1}{h}[1 - 1] = 0.
\end{aligned}$$

□

1. $\frac{\partial E[f(\theta)|S]}{\partial S} = 0$ if $\frac{\partial f(\theta)}{\partial \theta} = 0 \ \forall \theta$
2. $\frac{\partial E[f(\theta)|S]}{\partial S} < 0$ if $\frac{\partial f(\theta)}{\partial \theta} < 0 \ \forall \theta$
3. $\frac{\partial E[f(\theta)|S]}{\partial S} > 0$ if $\frac{\partial f(\theta)}{\partial \theta} > 0 \ \forall \theta$

Proof. For simplicity, the proof is written for a scalar-valued S. Extending the argument to vector-valued S is possible but not particularly informative.

The first statement, $\frac{\partial E[f(\theta)|S]}{\partial S} = 0$ if $\frac{\partial f(\theta)}{\partial \theta} = 0$, follows directly from Lemma 7 and the assumption that $f(\theta)$ is constant $\forall \theta$.

To show that $\frac{\partial E[f(\theta)|S]}{\partial S} < 0$ if $\frac{\partial f(\theta)}{\partial \theta} < 0 \ \forall \theta$, without any loss of generality let $S^2 > S^1$. Pick $\bar{\theta}_1, \bar{\theta}_2 : \int_0^{\bar{\theta}_1} f_S(\theta|S^1)d\theta = \int$:

is a nonnegative remainder term is bounded by $\frac{1}{2}\bar{\ell}_1$. This leaves us with

$$\lim_{A\to\infty}\int_0^{\bar{\ell}_1} f(\ell)\{_{\mathcal{S}}\mathcal{S}$$

[Figure 16 about here.]

Although the time paths are similar, there are some interesting properties of the stochastic values and policies (such as Lemma 2) which are not captured by the deterministic ones.

9.2 Value and policy functions

In general, the algorithms I use to compute decentralized solutions under open access are simpler than those used to compute the planner's solutions. This is because open access simplifies the continuation value to the cost of launching a satellite. I use R for all simulations, parallelizing where possible.

I compute optimal value functions by alternating between value and policy function iteration with a version of the one-way multigrid approach described in Chow and Tsitsiklis (1991). The multigrid approach involves computing the value function first on a coarse grid, then progressively refining the grid for further computation while using the previous results as initial guesses. I use linear interpolation to fill in new elements of the initial guess when moving to a finer grid. For simplicity, I use the same number of grid points, with the same limits, over S and D. Algorithm 1 describes how I compute the policy and value functions for a given grid $(\text{grid}_S; \text{grid}_D; \text{grid}_\ell)$ and given initial guess $\text{guess}(S; D; \ell)$.

Algorithm 1: Value function iteration with policy evaluation

Set

$$W_0(S, D, \ell) = guess(S, D, \ell)$$

for all $(S, D, \ell) \in (grid_S, grid_D, grid_\ell)$

Set $i = 1$ and $d = 100$ (some large value to begin).

while $d > \epsilon$ **do**

At each grid point in $(grid_S, grid_D, grid_\ell)$, use a numerical global optimizer to obtain

$$X^* = \arg\max_X \{pS - FX + \beta \hat{W}_{i-1}(S', D', E[\ell' | S', D'])\};$$

saves computation time for grids where the converged value function will only be used as an initial guess.[26] I set $\bar{d} = 1$ for all grids, and use a value for T between 10 and 100 depending

Algorithm 2: Open access launch and removal plans

1 At each point on the final grid used in the planner's solution, use a numerical global optimizer to obtain

$$R_i^o = \operatorname{argmax}_{R_i} \{ p - cR_i + E[\ell | S, D - SR_i] F \};$$

and set $R^o = SR_0^o$.

Algorithm 3: Stochastic open access launch time path

---

for t in $1, \dots, T-1$ **do**
  Draw ℓ_t from $\text{Bin}(\text{floor}(S_t), \min\{aS_t^2 + bS_tD_t, 1\})$
  Use a numerical root finder to find the X_t^o which solves

$$E_t[\ell_{t+1} | S_{t+1}, D_{t+1}] = r_s - r;$$

  using the laws of motion for S_t, D_t, and the form of $E[\ell | S, D]$.
  Update S_{t+1} and D_{t+1} using their laws of motion.
end
Set $X_T^o = 0$.

Simulating time paths for the planner, or even open access time paths with removal, is slightly more complicated due to the nature of the stochastic process for collisions. As mentioned above, the dynamics of the satellite and debris stocks make the process for ℓ_t dependent and heterogeneously distributed over time. Open access time paths with debris removal, and the planner's time paths generally, depend on future values of choice variables (R_{t+1} for open access, and (X_{t+1}, R_{t+1}) for the planner), which in turn depend on future draws. Simulating these stochastic processes directly is computationally challenging even in the open access case. Instead, I simulate the analogous deterministic processes. Algorithm 4 describes how I compute the deterministic time path of open access launch rates and cooperative removal.

Algorithm 4: Deterministic open access launch time path with cooperative endogenous debris removal

Set $T = 100$ (some large value). Initialize $\{X_t^1, R_t^1\}_0^T$.

Set $i = 1$ and $d = 100$ (some large value to begin).

while $d < \epsilon$ **do**

  Compute the launch rate sequence:

  for t in $1, \ldots, T-1$ **do**

    Use a numerical root finder to find the X_t^{i+1} which solves

$$E_t[\ell_{t+1}|S_{t+1}, D_{t+1} - R^i_{t+1}] = r_s - r;$$

    using the laws of motion for S_t, D_t, and the form of $E[\ell|S,D]$.

    Update S_{t+1} and D_{t+1} using their laws of motion.

  end

  Set $X_T^{i+1} = 0$.

  Compute the removal rate sequence:

  Use a numerical global optimizer to find the $\{R_t^{i+1}\}_0^T$ which maximizes

$$W_i^T(S,D;\ell) = \sum_{t=1}^{T} \beta^{t-1}(pS_t - FX_t^{i+1} - c_t R_t^{i+1})$$

  using the laws of motion for S_t, D_t, and the form of $E[\ell|S,D]$.

  $d \quad {}^{1}$

10 Tables and gures

Figures in the main text

Figure 1: Orbits of 56 cataloged satellites with mean altitudes between 700-710km. Source: Johnson (2004).

Table 1: Currently-operational satellites by origin, orbit class, and orbit type as of April 30, 2018

Breakdown of operating satellites					Total
by Country of origin	United States: 859	Russia: 146	China: 250	Other: 631	1,886
by Orbit Class	LEO: 1,186	MEO: 112	Elliptical: 40	GEO: 548	1,886
Breakdown of US satellites by Owner Type	Civil: 20	Commercial: 495	Government: 178	Military: 166	859

Source: Union of Concerned Scientists (2018).

Figure 2: Trends in orbit use.

Upper left panel: Number of active satellites in orbit per year since 2005.

Upper right panel: Monthly tracked non-spacecraft debris. These do not include derelict satellites which were not deorbited.

Lower left panel: Her ndahl-Hirschman Index for commercial launch services to low-Earth orbit and geostationary orbit.

Lower right panel: Evolution over time of the spatial distribution of ECOB collision risk index in low-Earth orbit. The large spike between 500-1000km is driven by a combination of commercial activity and China's 2007 anti-satellite missile test.

Sources: Union of Concerned Scientists (2018), NASA Orbital Debris Program Of ce (2017), and Letizia, Lemmens, and Krag (2018).

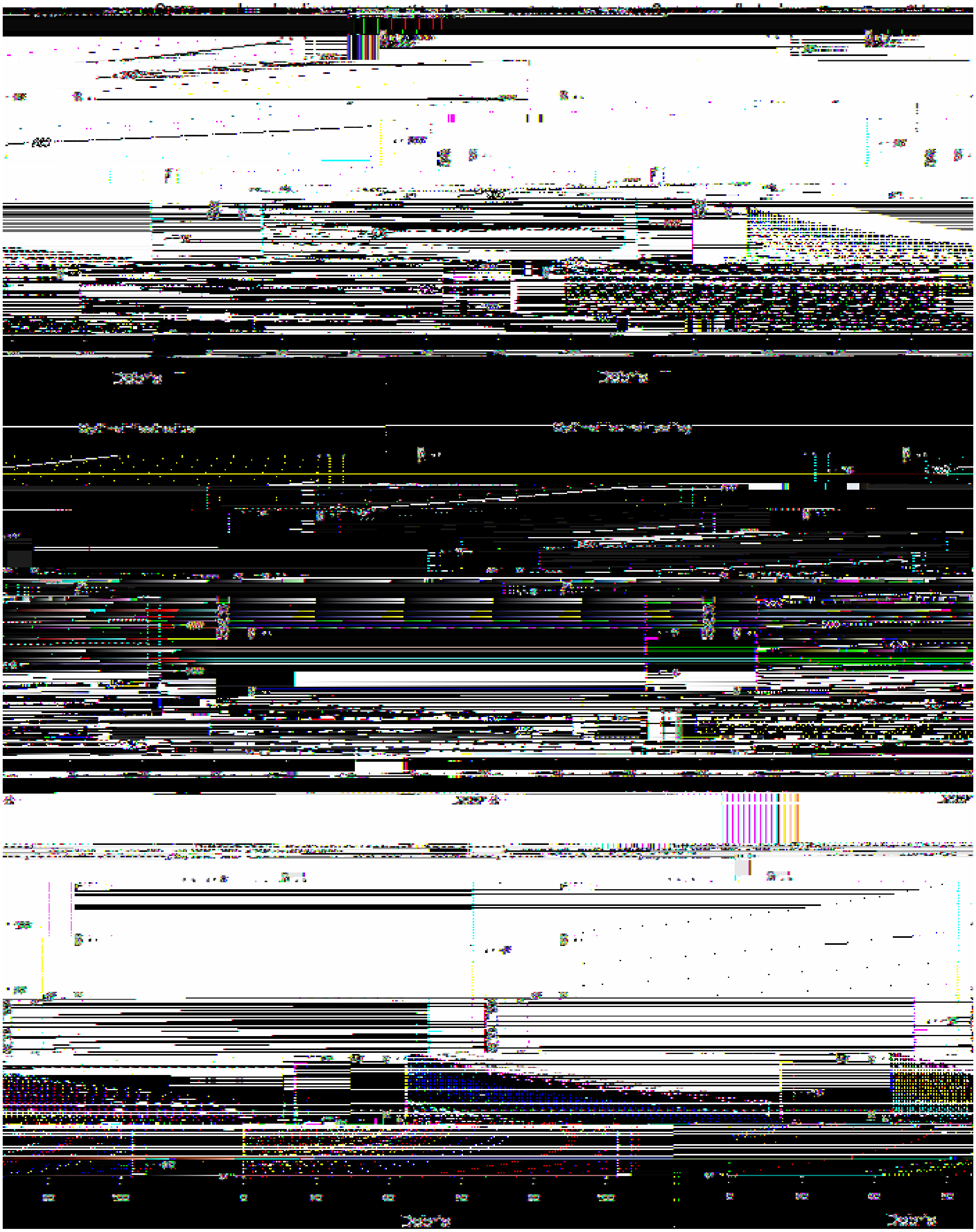

Figure 3: An example of the gap between open access and optimal launch policies, with the corresponding gap in fleet values.
The planner launches fewer satellites in every state than open access firms would. The value gap is maximized when (a) there is no debris and (b) the planner would stop launching satellites but open access firms do not.

Table 2: Examples of different types of orbit management policies

	Quantity control	Price control
Flow control	Launch permits	Launch taxes

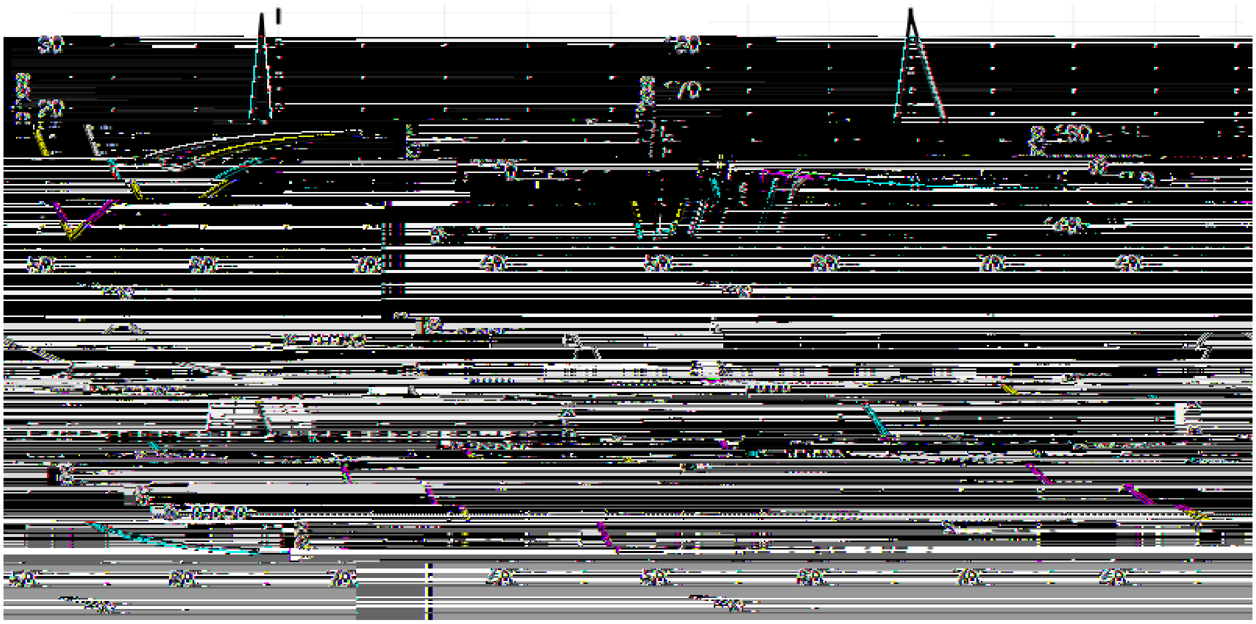

Figure 5:

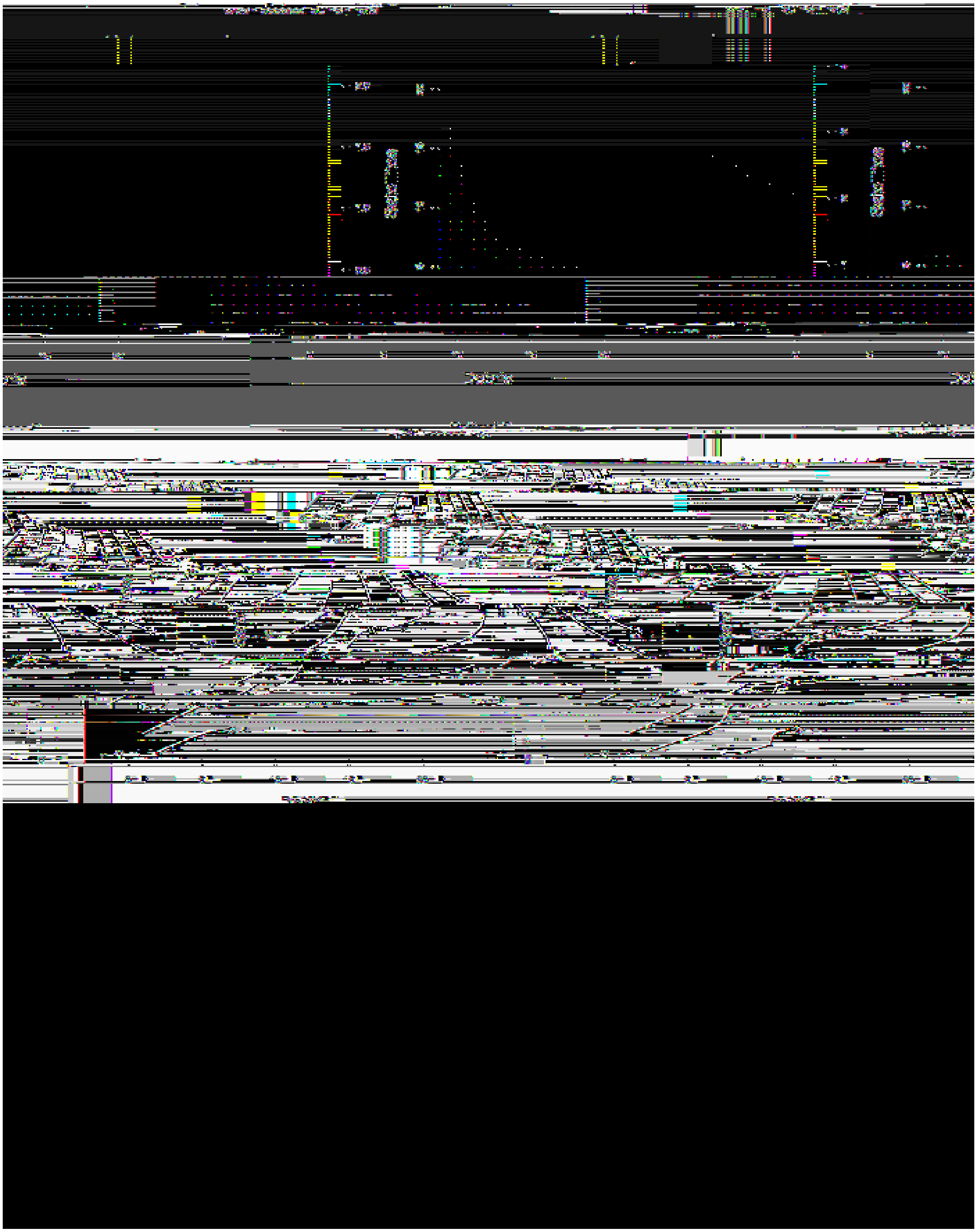

Figure 6: Optimal and open access stocks and launch rates.
Left column: Optimal launch rate (X_t), next-period satellite stock (S_{t+1}), and next-period debris stock (D_{t+1}).
Right column: Optimal launch rate (X_t), next-period satellite stock (S_{t+1}), and next-period debris stock (D_{t+1}).
The per-period return on a satellite is normalized to 1, the discount factor is set to 0.95, and the launch cost is set to 10. The open access next-period satellite stock is small but not zero in the upper right of the gure, while the optimal next-period satellite stock is zero there.

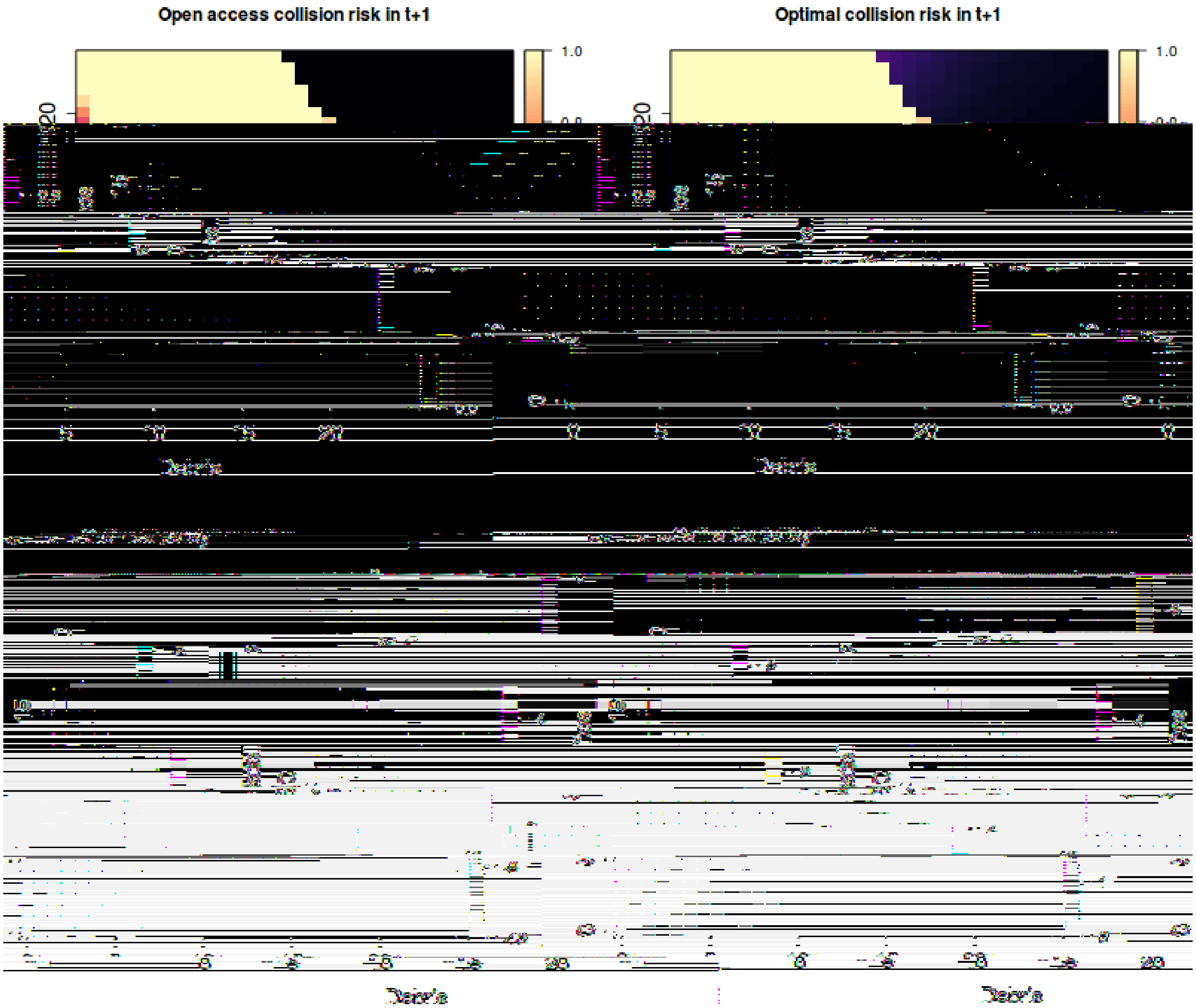

Figure 7: Optimal space traffic control policies.
Upper panels: The collision risk in $t+1$ ($E_t[\ell_{t+1}]$) under the optimal launch plan (left) and open access launch plan (right).
Lower left panel: An optimal satellite tax (stock control) in $t+1$.
Lower right panel: An optimal launch tax (flow control) in $t+1$. The tax in period t is normalized to 0.
The tax rates should be read as multiples of the per-period satellite return (normalized to 1). The white areas in the launch tax are where the collision risk is 1 and the tax is undefined; see Lemma 4 for an explanation of this feature. The collision risk jumps from 1 to 0 in the upper right section of the figures because there are no satellite left to be destroyed; see Figure 6 for the underlying satellite and debris stocks and launch rates. The marginal external cost is computed as $E_t[\xi(S_{t+1}, D_{t+1})] = E_t[\ell_{t+1}|\text{open access}] - E_t[\ell_{t+1}|\text{optimal}]$, following equations 9 and 13. The tax rates are then computed according to equations 31 and 32. The jump in the tax rates in the upper right is due to the slight gap in the satellite stocks described in Figure 6.

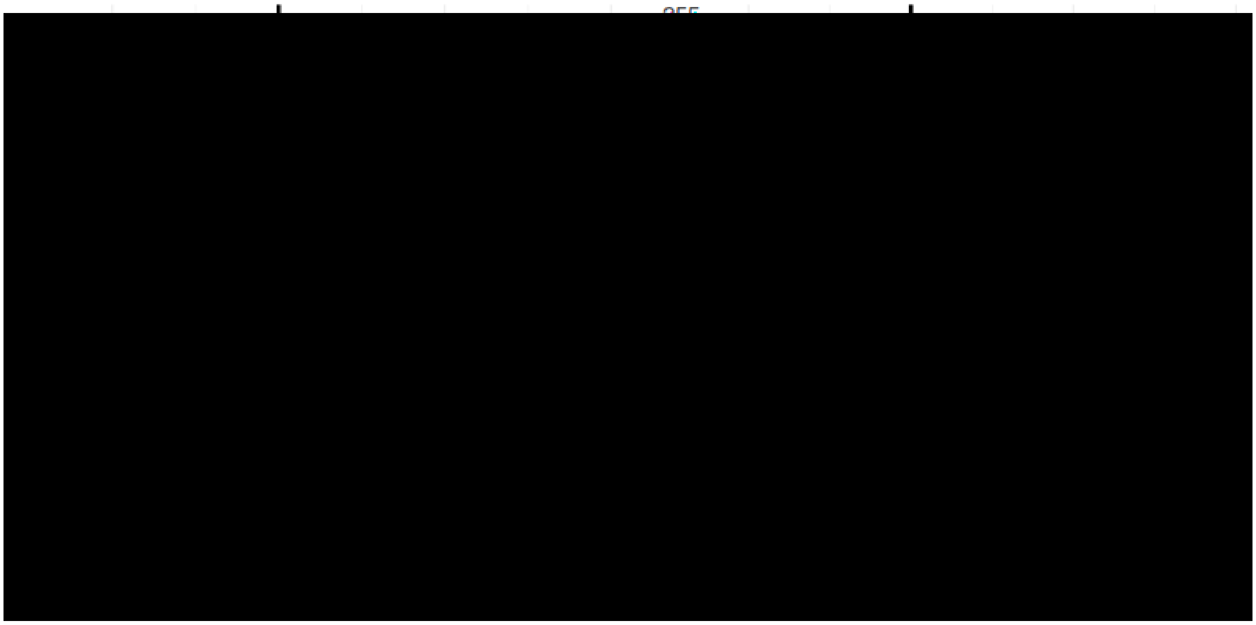

Figure 8: The effects of exogenous removal for free (black line) or a mandatory fee (blue line). When debris removal is provided to satellites owners for free, potential launchers respond by launching more satellites - even though the debris stock falls, the equilibrium collision risk remains unchanged. The equilibrium collision risk will fall when active debris removal is an option if and only if it is costly to satellite owners. In the case of costly debris removal, the launch rate falls to zero until the expected collision risk is no longer above the new equilibrium level. The dashed red line shows the equilibrium collision risk under open access.

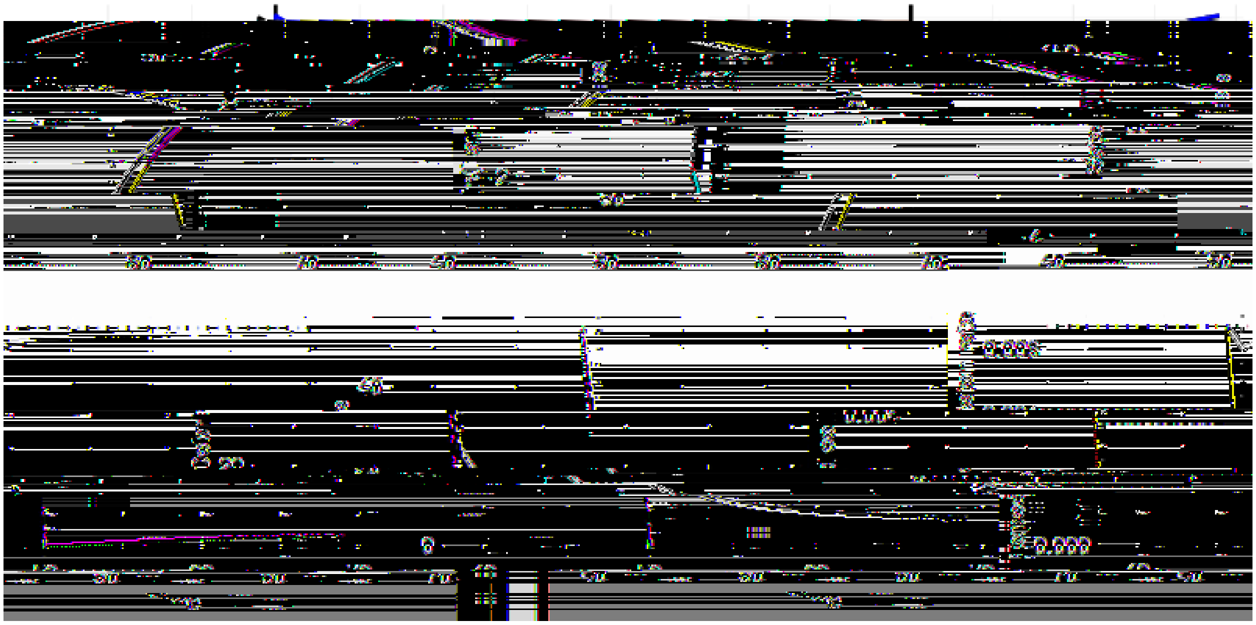

Figure 9: The effects of endogenously chosen cooperative debris removal (blue line) and exogenous removal for a mandatory fee (black line). The exogenous removal path in the exogenous case is set equal to endogenous removal path. Endogenous removal reduces both the equilibrium collision risk and the debris stock more effectively than exogenous removal, even if the same removal schedule is used. The endogenous removal schedule and launch response involves completely cleaning the orbit initially, and keeping the orbit relatively clean after. The same removal schedule provided exogenously induces rms to launch earlier than they would if they chose the schedule. The dashed red line shows the equilibrium collision risk under open access.

Figure 10: Comparing optimal and open access-cooperative launch and removal plans.
Upper row: The open access launch plan (left), cooperative removal plan (middle), and resulting eet value (right). The jump in the launch plan just above 10 re ects open access launches taking advantage of debris removal beginning, as shown in the time paths in Figure 9.
Middle row: The optimal launch plan (left), optimal removal plan(middle), and resulting eet value (right).
Bottom row: The gap between optimal plans/values and open access-cooperative plans/values. The gap between optimal and open access-cooperative eet values is maximized when (a) the planner would begin removing debris but cooperative satellite owners have not, and (b) just before open access launchers begin to launch again (anticipating removal) and the planner has stopped.

Figures in the appendices

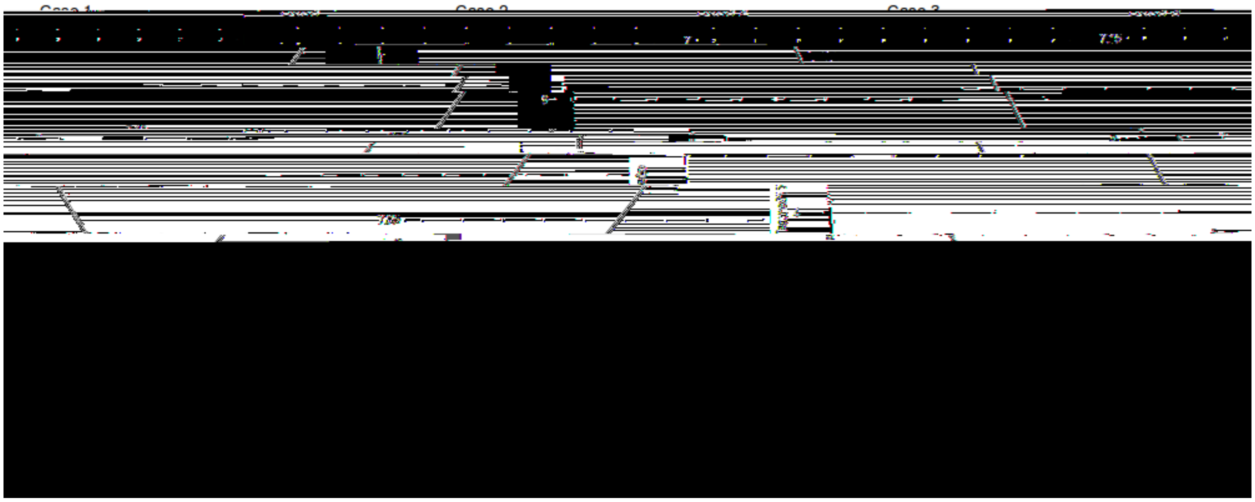

Figure 11: Three ways the open access launch rate may respond to collision risk draws.
Left panel: The open access launch rate is decreasing then increasing in the collision risk draw. The effect of additional debris dominates the launch decision when the collision risk draw is low, and the effect of fewer satellites dominates when the collision risk draw is high.
Middle panel: The open access launch rate is uniformly decreasing in the collision risk draw. The new-debris effect dominates for all draws.
Right panel: The open access launch rate is uniformly increasing in the collision risk draw. The fewer-satellites effect dominates for all draws.
The left panel and middle panel cases are more plausible than the right panel case under realistic relative orders of magnitude between the number of fragments created by satellite-satellite, satellite-debris, and debris-debris collisions.

Figure 12: Two collision rate functions and the private marginal bene t of debris removal
Upper row: Collision risks given different levels of debris removal.
Lower row: Private marginal bene ts of debris removal.
Left column: Negative exponential collision rate (globally concave).
Right column: Sigmoid collision rate (convex then concave).
Darker colors correspond to fewer satellites. More satellites may reduce or increase the marginal bene ts of debris removal, depending on whether satellites and debris are complements or substitutes in collision production.
Not shown: More initial debris in orbit shifts the removal bene t curves to the right. This makes the optimal removal amount increase until a jump to zero removal.

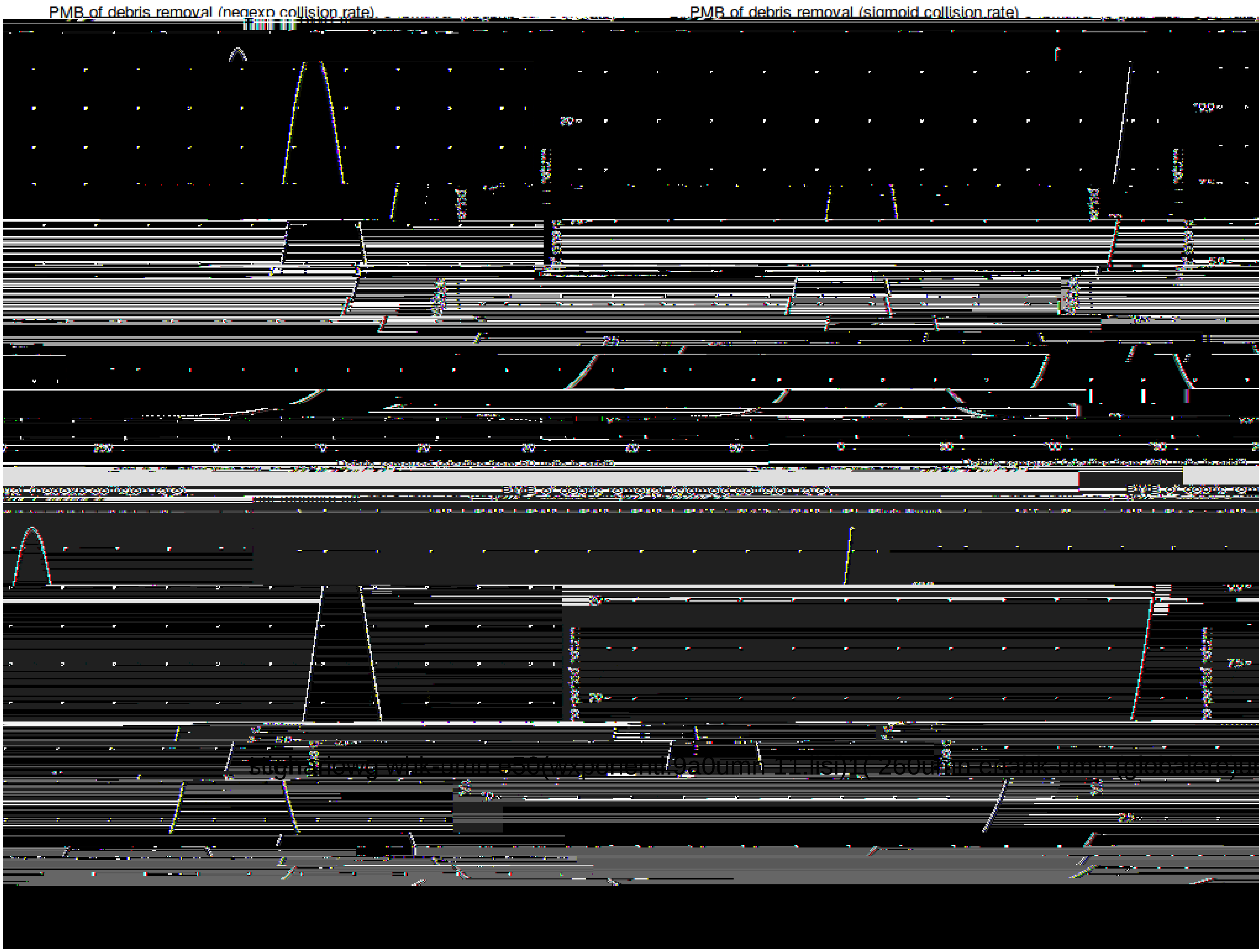

Figure 13:Nonconvexity and privately optimal removal
Upper row: High cost scenario where zero remov5move8orocostvHigh cost Upper row:

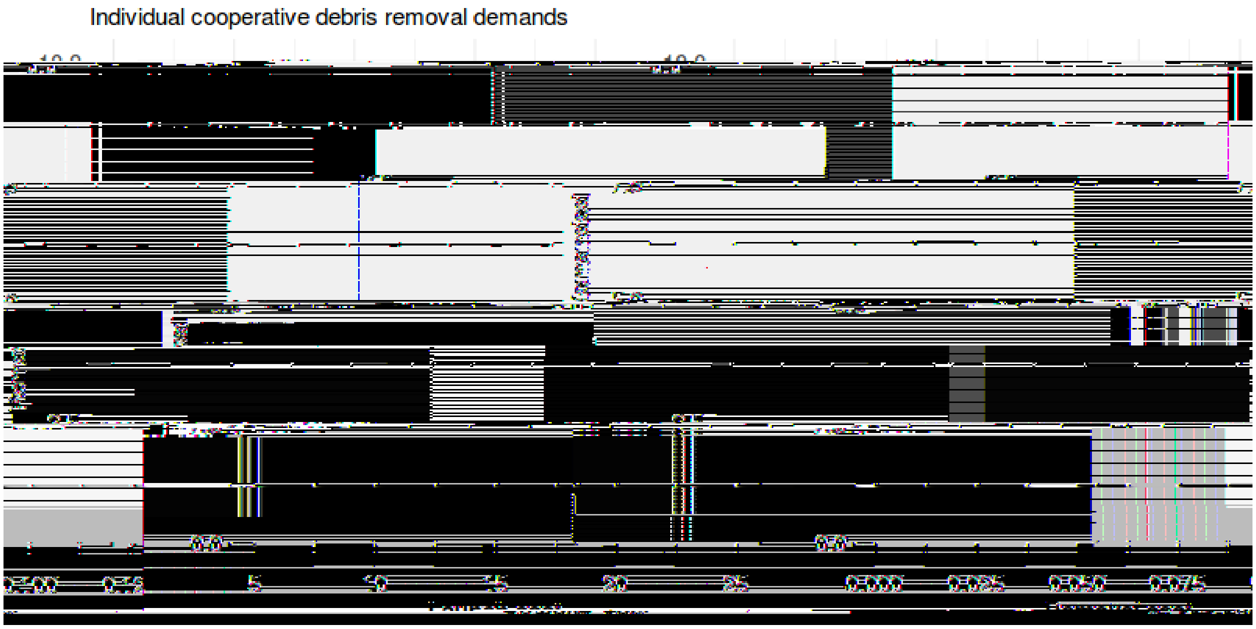

Figure 14:

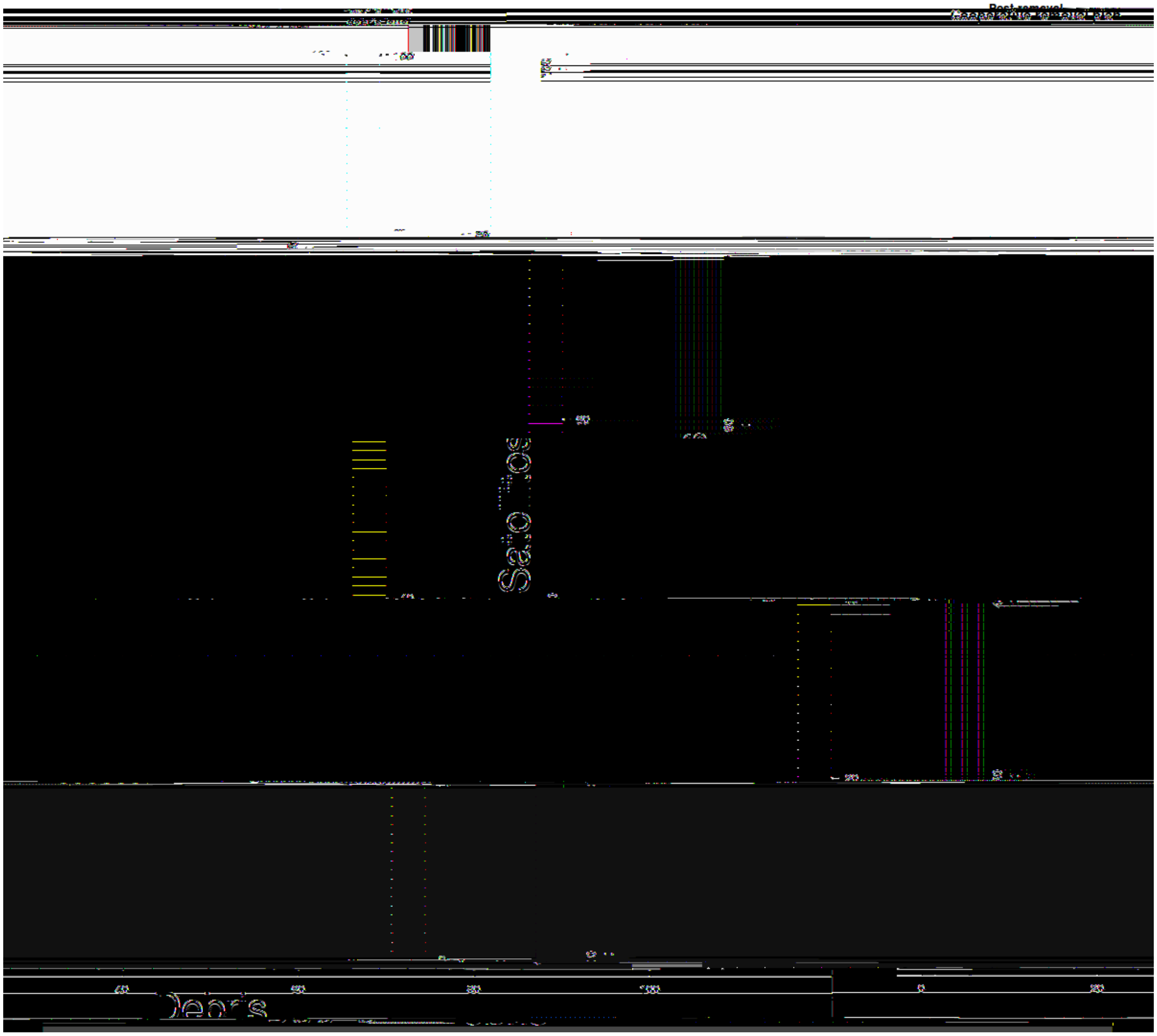

Figure 15: The effects of changes in the number of rms and debris in orbit on the post-removal level of debris.

The color scale represents the amount of debris left in orbit after removal. The cooperatively optimal post-removal level of debris does not depend on the amount of debris initially in orbit, but on the number of rms who are available to share the cost of removal. Once there are enough rms to begin removal the post-removal debris level is constant (full removal).

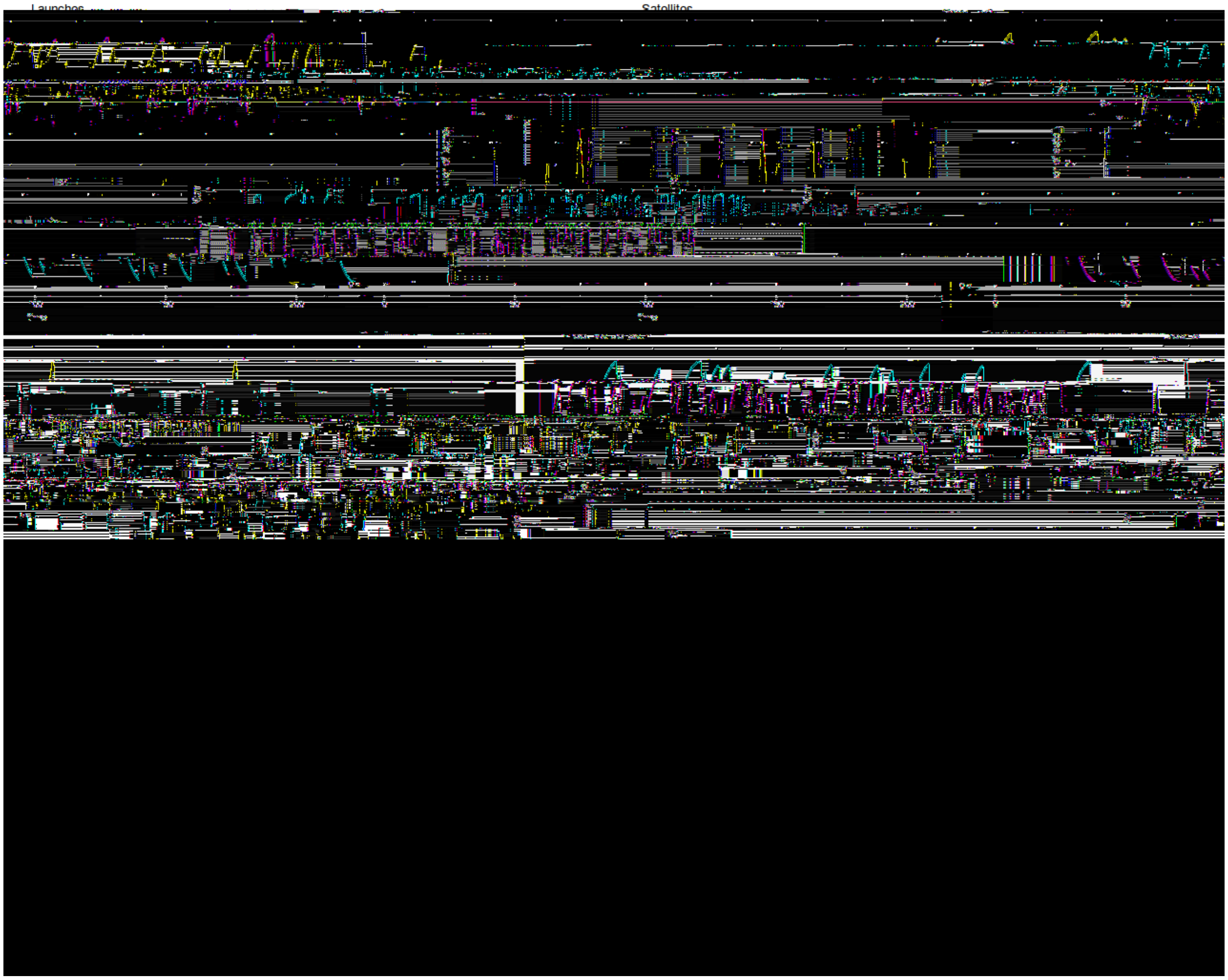

Figure 16:Time paths under the stochastic (blue line) and deterministic (black line) models. The red dots in the “collision rates” panel are the draws of ξ_t.